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# Nucleon spin sum rules: higher twist coefficients extraction

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We present the analysis of the lower moment of spin-dependent proton structure function up to $N^3LO$ order which is now available in the theoretical treatment of QCD correction. By using the standard perturbation theory, PT, and the modified perturbation theory with correct analytic properties, APT, we extract values of higher-twist coefficients and of the singlet axial charge from fresh low-energy Jefferson Lab data. We find that the analytical approach provides a better convergence of the higher twist and higher order corrections series than the standard PT description. We demonstrate that the use of the APT allows to achieve a good quantitative description of the Jefferson Lab data down to the momentum transfer of about the QCD scale parameter.

## 1. Introduction

The lepton Deep Inelastic Scattering (DIS) has been since long time a powerful tool to probe the structure of hadrons at small and intermediate scales. After the discovery of the parton structure of nucleons, DIS remains to be the primary source of experimental information on the distribution of quark and gluon fields in the nucleon and a valuable tool to test predictions of QCD, in particular on the nucleon spin structure.

Higher twist parameters are important ingredients of the nucleon spin structure [1]. These parameters arise from moment $\Gamma_1^p$ expansion to a Operator Product Expansion (OPE) series. HT extraction from experimental studies is relatively complicated as they are most pronounced at low momentum transfer $Q$. Although in this region very accurate Jefferson Lab (JLab) data [2] on moment $\Gamma_1^p$ are now available, but in this region HT contributions are shadowed by unphysical singularities of QCD coupling. This problem may be solved by the use of singularity-free couplings which allowed quite accurate extraction of higher twist and fairly well description of data down to rather low $Q$ [3, 4]. To avoid completely the issue of the unphysical singularities at $Q = \Lambda_{QCD} \sim 400\,\mathrm{MeV}$, strongly affecting the results of the analysis, we deal with the ghost-

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free analytic perturbation theory (APT) [5], which recently proved to be an intriguing candidate for a quantitative description of light quarkonia spectra within the Bethe-Salpeter approach (for a review on APT concepts and algorithms, see Ref. [6]).

Global analysis of the proton spin sum rule up to third order in the $\alpha_S$ for both the PT and the APT methods was made in Ref. [4]. Here we continue this investigation increasing up to the fourth-loop level. By using the recent JLab data on $\Gamma_1^p$ at $0.05 < Q^2 < 3.2\,\mathrm{GeV}^2$ [2], we extract values of higher twist coefficients and the value of the singlet axial charge by using expression for the N$^3$LO nonsinglet contribution to the proton spin sum rule [7]. This allows us to refine the results of HT extraction.

## 2. The lower moment of spin-dependent proton structure function $g_1^p$

The lower Cornwall-Norton moment of spin-dependent proton structure function $g_1^p$ is defined as follow

$$\Gamma_1^p(Q^2) = \int_0^1 dx\, g_1^p(x, Q^2)\,, \tag{1}$$

with $x = Q^2/2M\nu$, the energy transfer $\nu$ and the nucleon mass $M$. At large $Q > \Lambda_{QCD}$ , the moment $\Gamma_1^p(Q^2)$ is given by the OPE series in powers of $1/Q^2$ with the expansion coefficients related to nucleon matrix elements of operators of a definite twist (defined as the dimension minus the spin of the operator), and coefficient functions in the form of perturbative QCD series in $\alpha_s^n$. The total expression for the $\Gamma_1^p(Q^2)$ including the HT contributions reads

$$\Gamma_1^p(Q^2) = \frac{1}{12}\left[\left(a_3 + \frac{1}{3}a_8\right)E_{NS}(Q^2) + \frac{4}{3}a_0^{inv}\,E_S(Q^2)\right] + \sum_{i=2}^{\infty}\frac{\mu_{2i}^p}{Q^{2i-2}}, \tag{2}$$

where the triplet and octet axial charges are $a_3 \equiv g_A = 1.267\pm 0.004$ [8] and $a_8 = 0.585\pm 0.025$ [9], respectively, $\mu_{2i}^p$ - higher twist coefficients. $E_{NS}$ is the nonsinglet Wilson coefficient calculated as series in powers of $\alpha_s$ up to N$^3$LO order [7]

$$E_{NS}(Q^2) = 1 - \frac{\alpha_s}{\pi} - d_1\left(\frac{\alpha_s}{\pi}\right)^2 - d_2\left(\frac{\alpha_s}{\pi}\right)^3 - d_3\left(\frac{\alpha_s}{\pi}\right)^4 - O(\alpha_s^5)\,, \tag{3}$$

where for $n_f = 3$ in the MS scheme coefficients $d_1 = 3.583$ [10], $d_2 = 20.215$ [11], coefficient $d_3 = 175.7$ recently obtained in Ref. [7]. As for the singlet axial charge $a_0$, it is convenient to work with its RG invariant definition in the MS scheme $a_0^{inv} = a_0(Q^2 = \infty)$, in which all of the $Q^2$ dependency is factorized into the definition of the singlet Wilson coefficient $E_S(Q^2)$ [12]

$$E_S(Q^2) = 1 - \frac{\alpha_s}{\pi} - 1.096\left(\frac{\alpha_s}{\pi}\right)^2 - O(\alpha_s^3)\,. \tag{4}$$

It is clearly that the low-energy behavior of Wilson coefficients determined by infrared behavior of the strong running coupling $\alpha_S$. In our case (N$^3$LO) the expression for $\alpha_S$ [13] is given by

$$\begin{aligned}\alpha_s^{(4)}(L) \;=\;& \frac{1}{\beta_0 L} - \frac{1}{\beta_0^3 L^2}\beta_1 \ln L + \frac{1}{\beta_0^3 L^3}\left(\frac{\beta_1^2}{\beta_0^2}(\ln^2 L - \ln L - 1) + \frac{\beta_2}{\beta_0}\right) \\ &+ \frac{1}{\beta_0^4 L^4}\left[\frac{\beta_1^3}{\beta_0^3}\left(-\ln^3 L + \frac{5}{2}\ln^2 L + 2\ln L - \frac{1}{2}\right) - 3\frac{\beta_1\beta_2}{\beta_0^2}\ln L - \frac{\beta_3}{2\beta_0}\right],\end{aligned} \tag{5}$$

where $L = \ln(Q^2/\Lambda^2_{QCD})$ and $\beta$-function coefficients $\beta_0 = (33 - 2n_f)/12\pi$, $\beta_1 = (153 - 19n_f)/24\pi^2$, $\beta_2 = (77139 - 15099n_f + 325n_f^2)/3456\pi^3$, $\beta_3 = (29243 - 6946.3n_f + 405.089n_f^2 + 1.49931n_f^3)/256\pi^4$.

A detailed higher-twist analysis based on combined SLAC and JLab data (on proton, neutron $\Gamma_1^{p,n}(Q^2)$ [14] and nonsinglet $\Gamma_1^{p-n}(Q^2)$ moments [15]) was performed in Refs. [15–18] and recently in Refs. [3, 4]. In this paper we study the effect of higher loop corrections to the results of the description of $\Gamma_1^p(Q^2)$.

## 3. Higher twist coefficients extraction in PT

In the following when calculating the observables at any particular order of perturbation theory we will employ the prescription for the coefficient functions in the infrared region, where the order of power $\alpha_s$-series in coefficient functions is matched with the loop order in $\alpha_s$ itself. We extract the values of the higher-twist coefficients $\mu_{2i}^p$ and $a_0^{inv}$ by using expression (2) from very accurate JLab data [2]. The minimal borders of fitting domains in $Q^2$ are settled from the *ad hoc* restriction $\chi^2 < 1$ and monotonous behavior of the resulting fitted curves.

We start our analysis from PT case. Consider the best (1+3)-fit results on $\Gamma_1^p(Q^2)$ calculated at various PT orders (Table 1). Obviously that the leading singular behavior in the coefficient

Table 1. Dependence of the best (1+3)-fit results of $\Gamma_1^p(Q^2)$ data with NLO, N$^2$LO and N$^3$LO PT running coupling.

| Approach | $Q^2_{min}$, GeV$^2$ | $a_0^{inv}$ | $\mu_4^p/M^2$ | $\mu_6^p/M^4$ | $\mu_8^p/M^6$ |
|---|---|---|---|---|---|
| NLO | 0.416 | 0.37(2) | −0.031(3) | −0.021(4) | 0.016(1) |
| N$^2$LO | 0.416 | 0.26(8) | 0.036(3) | −0.11(4) | 0.057(1) |
| N$^3$LO | 0.591 | 0.03(18) | 0.020(9) | −0.44(13) | 0.29(5) |

function $\sim \ln^n L/L^m$ when $L \to 0$ comes from the highest power of $\alpha_s$. So in the infrared domain the influence of singularities gets stronger at higher orders of perturbation theory that may affect the data analysis below $1\,\mathrm{GeV}^2$. This fact also explains the behavior of the curves in Fig. 1, which presents the best (1+3)-fit results

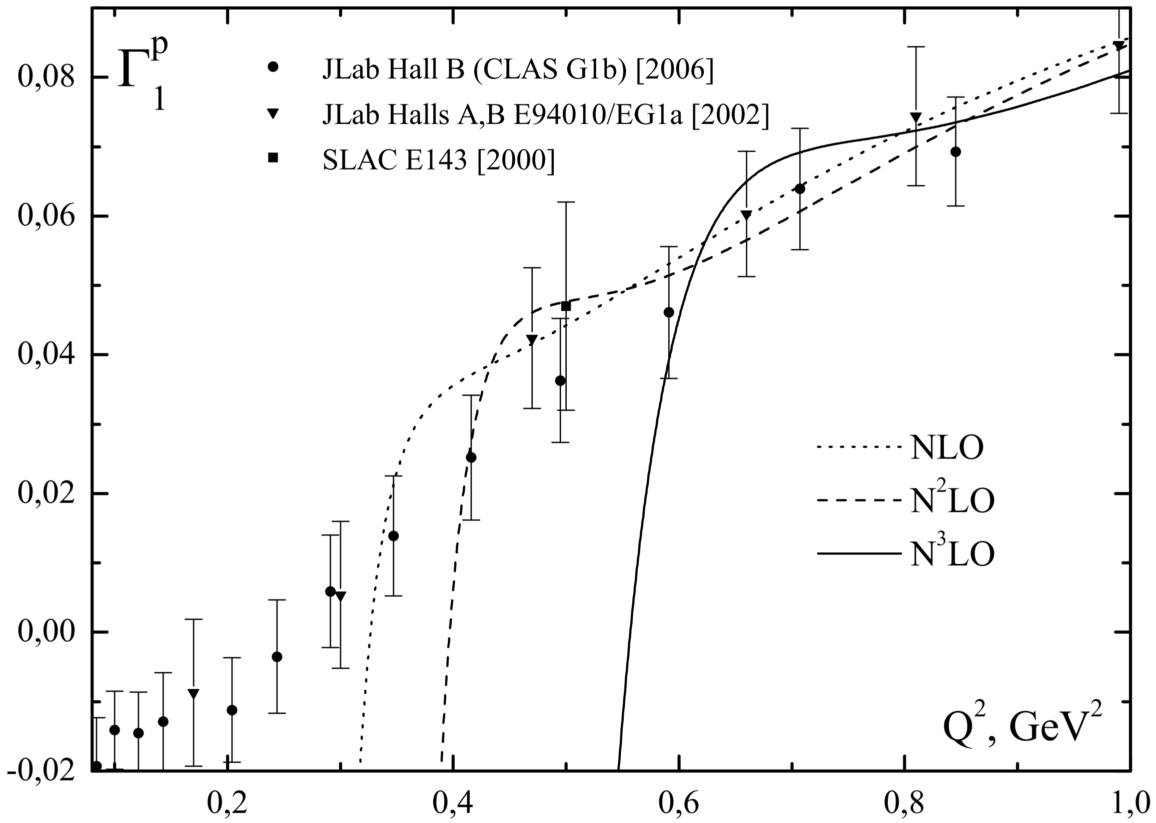


FIG. 1. Best (1+3)-parametric fits of JLab and SLAC data on $\Gamma_1^p(Q^2)$ calculated at various PT orders.

From Fig. 1 one can see that the higher PT orders yield a worse description of the proton sum rule data in comparison with the lowest orders. Such picture for precise JLab data on $\Gamma_1^p(Q^2)$ [2] implying the asymptotic character of the series in powers of perturbative $\alpha_s$.

One may ask to what extent these results are affected by the unphysical singularities when approaching to $Q \sim \Lambda_{QCD}$ in PT series for $\Gamma_{1,PT}^p$. Their influence becomes essential at $Q < 1\,\mathrm{GeV}$ where the HT terms starts to play an important role. The minimal border of the fitting domain $Q_{min}$ is tightly connected with the value of $\Lambda_{QCD}$, i.e. it is a scale, below which the influence of the ghost singularities becomes too strong and destroys the fit. To see how the $Q^2_{min}$ scale and fit results for $\mu^p$-terms change with varying $\Lambda_{QCD}$, we have done three different N$^3$LO fits with $\Lambda^{(4)}_{QCD} = 300,\, 400,\, 500$ MeV (see Table 2). It turns out that $a_0$ and HT terms are quite sensitive to the unphysical singularity position, and their values noticeably varies with $\Lambda_{QCD}$. The existence of unphysical singularities substantially complicates the analysis of low-energy data. The standard PT approach does not yield a stable information on the nonperturbative parameters. The APT model is free of such a problem thus providing a reliable tool to investigate

Table 2. Dependence of the best $(1+3)$-parametric fit results of $\Gamma_1^p(Q^2)$ data on $\Lambda_{QCD}^{(4)}$ in PT.

| $\Lambda_{QCD}$, MeV | $Q^2_{min}$, GeV$^2$ | $a_0^{inv}$ | $\mu_4^p/M^2$ | $\mu_6^p/M^4$ | $\mu_8^p/M^6$ |
|---|---|---|---|---|---|
| 300 | 0.416 | 0.22(9) | 0.04(4) | −0.13(4) | 0.07(1) |
| 400 | 0.707 | −0.1(3) | −0.3(2) | −0.8(3) | 0.5(1) |
| 500 | 1.2 | −0.1(6) | 0.4(5) | −1.4(11) | 1.7(8) |

the behavior of HT terms extracted directly from the low energy data [4]. This provides a motivation for the analysis performed with using APT.

## 4. Analytic approach

The moments of the structure functions are analytic functions in the complex $Q^2$-plane with a cut along the negative real axis, as has been demonstrated in Ref. [19]. On the other hand, the standard PT approach does not support these analytic properties. The influence of requiring these properties to hold in the deep-inelastic scattering description has been studied previously by I.L. Solovtsov and D.V. Shirkov in Refs. [20]. Here, we continue this investigation, by applying the APT method, which gives the possibility of combining the renormalization group resummation with correct analytic properties of the QCD corrections, to the low energy data on proton spin sum rule $\Gamma_1^p(Q^2)$.

In the framework of the analytic approach we can write the expression for $\Gamma_1^p(Q^2)$ in the form

$$\Gamma^p_{1,APT}(Q^2) = \frac{1}{12}\left[\left(a_3 + \frac{1}{3}a_8\right)E_{NS}^{APT}(Q^2) + \frac{4}{3}a_0^{inv}\, E_S^{APT}(Q^2)\right] + \sum_{i=2}^{\infty}\frac{\mu_{2i}^p(Q^2)}{Q^{2i-2}}, \qquad (6)$$

which is analogous to one in the standard PT (2). Wilson coefficients in APT have the same form as in PT. But the expansion is now performed not in the powers of $\alpha_S$, but in functions $\mathcal{A}_k$, which have analytic properties:

$$E_{NS}^{APT}(Q^2) = 1 - \frac{\mathcal{A}_1(Q^2)}{\pi} - d_1\frac{\mathcal{A}_2(Q^2)}{\pi^2} - d_2\frac{{}_3(Q^2)}{\pi^3} - d_3\frac{\mathcal{A}_4(Q^2)}{\pi^4}\,, \qquad (7)$$

$$E_S^{APT}(Q^2) = 1 - 0.318\,\mathcal{A}_1^{(4)}(Q^2) - 0.111\,\mathcal{A}_2^{(4)}(Q^2)\,, \qquad (8)$$

where $\mathcal{A}_k$ is the analyticized $k$-th power of PT running coupling in the Euclidean domain

$$\mathcal{A}_k(Q^2) = \frac{1}{\pi}\int_0^{+\infty}\frac{\mathrm{Im}([\alpha_s(-\sigma)]^k)\,d\sigma}{\sigma + Q^2}\,. \qquad (9)$$

It should be stressed that APT functions $\mathcal{A}_k$ are stable with respect to different loop orders at low energy scales $Q \lesssim 1$ GeV [6]. Therefore, APT method is not quite sensitive to the value of $\Lambda_{QCD}$. To test this fact, we fulfilled three different fits of the proton $\Gamma_1^p(Q^2)$ data with $\Lambda_{QCD}^{(4)} = 300,\ 400,\ 500$ MeV as we have done before in the standard PT. The result of these fits are shown in Table 3. Comparing these results with the data from Table 2, we see that corresponding results

Table 3. Sensitivity of the best N$^3$LO APT fit results of proton $\Gamma_1^p(Q^2)$ data to $\Lambda_{QCD}^{(4)}$ variations.

| $\Lambda_{QCD}$, MeV | $Q^2_{min}$, GeV$^2$ | $a_0^{inv}$ | $\mu_4^p/M^2$ | $\mu_6^p/M^4$ | $\mu_8^p/M^6$ |
|---|---|---|---|---|---|
| 300 | 0.071 | 0.36(3) | −0.067(4) | 0.009(1) | −0.0004(1) |
| 400 | 0.071 | 0.38(4) | −0.066(4) | 0.009(2) | −0.0005(1) |
| 500 | 0.071 | 0.40(3) | −0.066(3) | 0.009(1) | −0.0004(1) |

in the standard PT are much more sensitive to $\Lambda_{QCD}$-variations, than ones in APT. Consequently value of $\Gamma^p_{1,APT}(Q^2)$ is quite stable with respect to small variations of $\Lambda_{QCD}$, in contrast with huge instability of $\Gamma^p_{1,PT}$: it changes now by about $2-3\%$ within the interval $\Lambda_{QCD}^{(4)} = 300-500\,\mathrm{MeV}$. The same was previously observed for the Bjorken function $\Gamma^{p-n}_{1,APT}(Q^2)$ in Ref. [3]. Due to this fact the low-$Q$ data on $\Gamma_1^p(Q^2)$ cannot be used for a determination of $\Lambda_{QCD}$ in the APT approach.

Extending the analysis of Ref. [20] to lower $Q$ values, we estimated the relative size of APT contributions to $\Gamma_1^p(Q^2)$. It turned out that the third term $\sim \mathcal{A}_3$ contributes no more than 5% and fourth term $\sim \mathcal{A}_4$ contributes no more than 1% to the sum, thus supporting the practical convergence of the APT series. At the same time third and fourth terms of the PT contributions no more than 12% and 10%, respectively, which corresponds to a worse convergence of the perturbative series.

In Fig. 2, we show best fits of the combined data set for the function $\Gamma_1^p(Q^2)$ in the standard PT (NLO, N$^2$LO and N$^3$LO) and the APT (N$^3$LO) approaches.

In Table 4 we present the combined fit results of the proton $\Gamma_1^p(Q^2)$ data in different APT orders.

From Table 4 and Fig. 2 one can see that APT fits have loop stability. That is, the values of HT remain the constants in the different orders of the APT. Using the analytic approach is preferable because it allows extracting from experimental data a stable values of the nonperturbative parameters.

## 5. Summary

In this paper we have extracted values of $a_0$ and HT terms from very accurate JLab data on the first moment of spin structure function $g_1^p$ up to the fourth-loop level by using the standard PT and APT approaches. The experimental DIS data at very low $Q \sim \Lambda_{QCD}$ are usually dropped

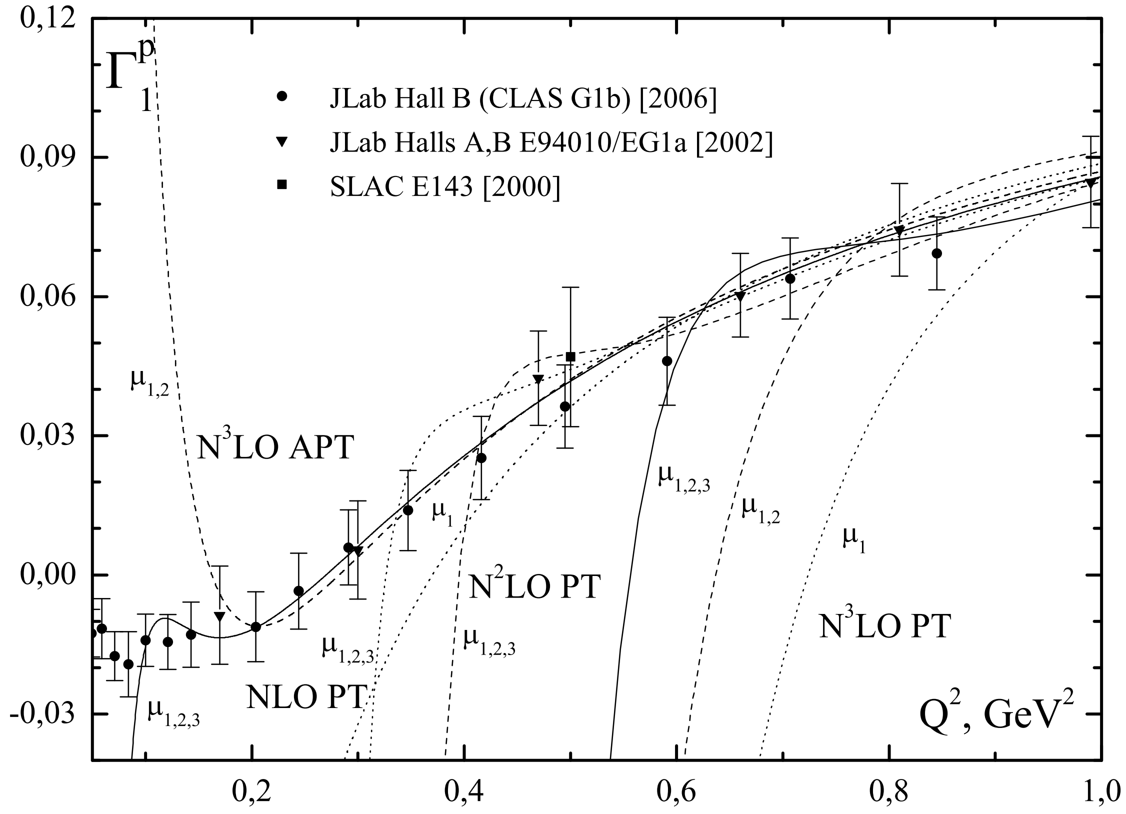


FIG. 2. Best (1+1,2,3)-parametric fits of the JLab and SLAC data on $\Gamma_1^p$.

Table 4. Combined fit results of the proton $\Gamma_1^p(Q^2)$ data in different APT orders.

| Approach | $Q^2_{min}$, GeV$^2$ | $a_0^{inv}$ | $\mu_4^p/M^2$ | $\mu_6^p/M^4$ | $\mu_8^p/M^6$ |
|---|---|---|---|---|---|
| NLO APT | 0.47 | 0.37(3) | $-0.056(4)$ | 0 | 0 |
| | 0.17 | 0.41(3) | $-0.069(4)$ | 0.008(1) | 0 |
| | 0.10 | 0.43(3) | $-0.076(4)$ | 0.013(1) | $-0.0006(1)$ |
| $N^2$LO APT | 0.47 | 0.37(4) | $-0.056(4)$ | 0 | 0 |
| | 0.17 | 0.41(3) | $-0.070(4)$ | 0.008(1) | 0 |
| | 0.10 | 0.43(3) | $-0.077(4)$ | 0.012(1) | $-0.0007(1)$ |
| $N^3$LO APT | 0.47 | 0.37(4) | $-0.057(4)$ | 0 | 0 |
| | 0.17 | 0.40(3) | $-0.070(4)$ | 0.008(1) | 0 |
| | 0.10 | 0.43(3) | $-0.077(4)$ | 0.013(1) | $-0.0007(1)$ |

from the analysis of $a_0$ and higher twists terms in the PT because of unphysical singularities. To get rid of this defect we use the APT model for the infrared-finite QCD coupling $\alpha_s$.

We performed a systematic comparison of extracted values of the HT terms in different orders of PT and APT and found again fundamental differences between these two approaches. In the APT approach the convergence of both the higher orders and HT series is much better. In the APT case, the subsequent terms (after twist-4 term) are essentially smaller and quickly decreasing than in the conventional PT. So, the APT absorbs some part of nonperturbative dynamics described by HT terms. It was shown that the satisfactory description of the proton spin sum rule data down to $Q \sim \Lambda_{QCD} \simeq 350$ MeV was achieved by taking the analytic higher twists and higher orders PT contributions into account simultaneously. Our analysis shown that the remarkable property of the APT approach create a basis for the its preferable application at low momentum transfer, $Q < 1\,\mathrm{GeV}$.

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