The Universe as a Nonuniform Lattice in the Finite-Dimensional Hypercube. II.Simple Cases of Symmetry Breakdown and Restoration

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This paper continues a study of field theories specified for the nonuniform lattice in the finite-dimensional hypercube with the use of the earlier described deformation parameters. The paper is devoted to spontaneous breakdown and restoration of symmetry in simple quantum-field theories with scalar fields. It is demonstrated that an appropriate deformation opens up new possibilities for symmetry breakdown and restoration. To illustrate, at low energies (far from the Plancks) it offers high-accuracy reproducibility of the same results as with a nondeformed theory. In case of transition from low to higher energies and vice versa it gives description for new types of symmetry breakdown and restoration depending on the rate of the deformation parameter variation in time, and indicates the critical points of the previously described lattice associated with a symmetry restoration. Besides, such a deformation enables one to find important constraints on the initial model parameters having an explicit physical meaning.

I. INTRODUCTION

This paper is written to continue a study presented in [1]. In the earlier work it has been shown that with the use of a new small parameter α,$0 < \alpha \leq 1/4$ and its statistical mechanics counterpart τ,$0 < \tau \leq 1/4$, resultant from the density matrix deformation in quantum and statistical mechanics of the Early Universe (Planck scale) and introduced by the author in his previous works forming a series [1]– [6], the Universe may be described as a nonuniform lattice in the hypercube with finite edges. It should be noted that a quantum theory naturally appears in this picture, the transition to higher energies being nothing else but the transition to lower lattice nodes. In [1], [3]one of the simplest transitions from the density matrix deformation (density pro-matrix) $\rho(\alpha)$ ($\rho(\tau)$ respectively) to deformation in a quantum field theory $\psi(\widetilde{\alpha}, x)$, or from Neumann to Schrodinger picture, may be found. The present paper is a study into a simple case of spontaneous symmetry breakdown for this deformed quantum theory with scalar fields. It is demonstrated that at lower energies far from the Plancks the results of the nondeformed case are reproduced with a very high accuracy. Compared to the nondeformed case, new results associated with symmetry breakdown and restoration appear on going from low to higher energies and vice versa due to the rate of the deformation parameter variation in time. The transition from high energies to lower ones is of particular importance, being closely associated with cosmological models [20], [21]. Points of the symmetry restoration are marked on the lattice $Lat_{\alpha}^{\mathcal{T}}$,introduced previously in [1], and referred to as the critical points. The constraints imposed on the deformation parameters lead to significant additional conditions for the initial model parameters having an explicit physical meaning.

The paper is structured as follows: section 1 is an introduction of the problem. Section 2 is a brief outline of the principal results obtained on the deformed density matrix in quantum mechanics $\rho(\alpha)$ and statistical mechanics of the Early Universe $\rho(\tau)$; transition to the associated deformation of a quantum field theory and deformed wave function $\psi(\widetilde{\alpha}, x)$, giving the basis for the description of lattice $Lat_{\alpha}^{\mathcal{T}}$. The section 3 presents the results obtained on spontaneous symmetry breakdown and restoration in model with scalar fields. And in Conclusion the main inferences of the work are summarized.

II. PROBLEM SURVEY AND NOTATION

Now it is commonly accepted that physics of the Early Universe [7] (scales on the order of the Plancks) should be distinct from the physics of the Universe in its current state. This may be inferred proceeding from different

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approaches, e.g., string theory [8], quantum gravics [9], [10], [11] and some others [12], [13]. And quantum mechanics of the Early Universe may be considered as a deformation of a well-known quantum mechanics. In this case the deformation is understood as an extension of a particular physical theory by the introduction of an additional parameter or several parameters in such a way that the initial theory appears in the limiting transition to certain fixed values of these parameters [14]. As distinct from the approach associated with the Heisenbergs algebra deformation that modifies the Heisenberg uncertainty relations [15], for example [9], [16], [13], [17]), in his publications [1]– [6] the author has approached the deformation of quantum mechanics of the Early Universe using radically new method - the density matrix deformation. In his approach the deformation parameter is a dimensionless value α: $\alpha = l_{min}^2/x^2$, where l_{min} is a minimal length and x is the measuring scale. Generally it is accepted that the value of l_{min} is on the order of the Plancks length $l_{min} \sim l_p$. However, this assumption is not a must. A new deformed object $\rho(\alpha)$, referred to as a **density pro-matrix** possesses a number of interesting features described in [2], [3], giving in the low-energy limit an ordinary quantum-mechanical density matrix. Moreover, $\rho(\alpha)$ allows for a new approach to the solution of some problems: Liouville equation deformation in the Early Universe and close to the black hole [3], black hole entropy and its quantum corrections [1], [3]– [5],entropy density [3]– [5], information loss problems in the processes associated with black holes [4], [5].

We are especially interested in the following feature of $\rho(\alpha)$:

$$Sp[\rho(\alpha)] - Sp^2[\rho(\alpha)] \approx \alpha. \tag{1}$$

From whence it follows that the value for $Sp[\rho(\alpha)]$ satisfies the above-stated condition

$$Sp[\rho(\alpha)] \approx \frac{1}{2} + \sqrt{\frac{1}{4} - \alpha} \tag{2}$$

and therefore $0 < \alpha \leq 1/4$.

Besides, solution (1) will be further used as an exponential ansatz [2], [3]:

$$\rho^*(\alpha) = \sum_i \alpha_i exp(-\alpha)|i><i|, \tag{3}$$

where all $\alpha_i > 0$ are independent of α and their sum is equal to 1. Then

$$Sp[\rho^*(\alpha)] = exp(-\alpha). \tag{4}$$

In [6] it has been demonstrated that statistical mechanics of the Early Universe should be also deformed compared to a well-known statistical mechanics associated with Gibbs distribution [18]. Similar to quantum mechanics, the principal object in this case is just the density matrix, though now statistical. Here the deformation parameter is $\tau = T^2/T_{max}^2$, varying in the same interval $0 < \tau \leq 1/4$ as α. A new object appearing as a result of this deformation $\rho_{stat}(\tau)$, that is called the statistical density pro-matrix, gives in the limit of temperatures far from maximum $T_{max} \sim T_p$ (where T_p is Plancks temperature)a well-known statistical density matrix ρ_{stat} [18] and satisfies the analog of conditions described by (1)-(4) with a change of α by τ [6], i.e.

$$Sp[\rho_{stat}(\tau)] - Sp^2[\rho_{stat}(\tau)] \approx \tau. \tag{5}$$

Correspondingly,

$$Sp[\rho_{stat}(\tau)] \approx \frac{1}{2} + \sqrt{\frac{1}{4} - \tau}. \tag{6}$$

Naturally, with an appropriate change exponential ansatz (3) is here also the case, and solution (5) satisfies the equation

$$Sp[\rho^*(\tau)] = exp(-\tau). \tag{7}$$

Taking into consideration duality of relation between time and temperature that follows from the Generalized Uncertainty Relations in quantum theory and thermodynamics [6], [19]:

$$\begin{cases} \Delta x & \geq \quad \frac{\hbar}{\Delta p} + \alpha' L_p^2 \frac{\Delta p}{\hbar} + ... \\ \\ \Delta t & \geq \quad \frac{\hbar}{\Delta E} + \alpha' t_p^2 \frac{\Delta E}{\hbar} + ... \\ \\ \Delta \frac{1}{T} & \geq \quad \frac{k}{\Delta U} + \alpha' \frac{1}{T_p^2} \frac{\Delta U}{k} + ..., \end{cases} \tag{8}$$

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In [1] it has been shown that the Universe may be represented as a four-dimensional lattice $Lat^{\mathcal{Z}}_{\alpha}$ in hypercube $I^4_{1/4}$ with edge $I_{1/4} = (0; 1/4]$, where an arbitrary point of $Lat^{\mathcal{Z}}_{\alpha}$ is characterized by coordinates $(\widetilde{\alpha}, \tau) = (\alpha_1, \alpha_2, \alpha_3, \tau)$, and $\alpha_1, \alpha_2, \alpha_3$ are the values of parameter α in three space dimensions. They are taken as independent of each other as, in principle, at very high energies (on the order of the Plancks) the space coordinates may be noncommutative [7], [10], [11]:

$$[x_i, x_j] \neq 0.$$

In this case, independently of each other, all the described variables may take on one and the same discrete and nonuniform series of values $1/4, 1/16, 1/36, 1/64, ...$

Since for exponential ansatz (3) a prototype of the pure state (wave function) in the introduced formalism is represented by the density pro-matrix

$$\rho^*(\alpha) = exp(-\alpha)|\psi><\psi|, \tag{9}$$

α-deformation of wave functions, i.e. fields, in QFT is of the following form [3]:

$$\psi(x) \mapsto \psi(\alpha, x) = \mid \theta(\alpha) \mid \psi(x), \tag{10}$$

where

$$\mid \theta(\alpha) \mid = exp(-\alpha/2) \tag{11}$$

or

$$\theta(\alpha) = \pm exp(-\alpha/2)(cos\gamma \pm isin\gamma). \tag{12}$$

Further in the text (section 3) it is assumed that

1) α of the exponential factor in formula (3) is the same for all space coordinates $\alpha_1, \alpha_2, \alpha_3$ of the point of lattice $Lat^{\mathcal{Z}}_{\alpha}$. This means that as yet the noncommutativity effect is of no special importance, and parameter α is determined by the corresponding energy scale;

2) parameter α is dependent on time only $\alpha = \alpha(t)$. This condition is quite natural since α plays a part of the scale factor and is most often dependent on time only (especially in cosmology) [20], [21]);

3) as all physical results should be independent of a selected normalization $\theta(\alpha)$, subsequent choice of the normalization will be

$$\theta(\alpha) = exp(-\alpha/2), \tag{13}$$

that is $\gamma = 0$.

III. SPONTANEOUS SYMMETRY BREAKDOWN AND RESTORATION IN A MODEL WITH SCALAR FIELDS

First, take a well-known Lagrangian for a scalar field [22]:

$$L = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2}\mu^2 \phi^2 - \frac{1}{4}\lambda \phi^4, \tag{14}$$

where $\lambda > 0$.

Because of the transformation in the above deformation

$$\phi \Rightarrow \phi(\alpha) = exp(-\frac{\alpha}{2})\phi, \tag{15}$$

where $\alpha = \alpha(t)$, α is a deformed Lagrangian of the form

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$$L(\alpha) = \frac{1}{2}(\partial_\mu \phi(\alpha))^2 + \frac{1}{2}\mu^2 \phi(\alpha)^2 - \frac{1}{4}\lambda\phi(\alpha)^4. \tag{16}$$

It should be noted that a change from Lagrangian to the Hamiltonian formalism in this case is completely standard [23] with a natural changing of ϕ by $\phi(\alpha)$. Consequently, with the use of a well-known formula

$$H(\alpha) = p\dot{q} - L(\alpha) \tag{17}$$

we obtain

$$H(\alpha) = \frac{1}{2}(\partial_0 \phi(\alpha))^2 + \frac{1}{2}\sum_{i=1}^{3}(\partial_i \phi(\alpha))^2 - \frac{1}{2}\mu^2 \phi(\alpha)^2 + \frac{1}{4}\lambda\phi(\alpha)^4. \tag{18}$$

We write the right-hand side of (18) so as to solve it for ϕ and α. As α is depending solely on time, the only nontrivial term (to within the factor of $1/2$) in the right-hand side of (18) is the following:

$$(\partial_0 \phi(\alpha))^2 = exp(-\alpha)(\frac{1}{4}\dot{\alpha}^2 \phi^2 - \dot{\alpha}\phi\partial_0 \phi + (\partial_0 \phi)^2)$$

.

Thus, α - deformed Hamiltonian $H(\alpha)$ is rewritten as

$$H(\alpha) = exp(-\alpha)[-\frac{1}{2}\dot{\alpha}\phi\partial_0 \phi + \frac{1}{2}\sum_{j=0}^{3}(\partial_j \phi)^2 + (\frac{1}{8}\dot{\alpha}^2 - \frac{1}{2}\mu^2)\phi^2 + \frac{1}{4}exp(-\alpha)\lambda\phi^4]. \tag{19}$$

To find a minimum of $H(\alpha)$, we make its partial derivatives with respect ϕ and α equal to zero

$$\begin{cases} \frac{\partial H(\alpha)}{\partial \phi} = exp(-\alpha)[-\frac{1}{2}\dot{\alpha}\partial_0 \phi \\ +(\frac{1}{4}\dot{\alpha}^2 - \mu^2)\phi + exp(-\alpha)\lambda\phi^3] = 0 \\ \frac{\partial H(\alpha)}{\partial \alpha} = -exp(-\alpha)[-\frac{1}{2}\dot{\alpha}\phi\partial_0 \phi + \frac{1}{2}\sum_{j=0}^{3}(\partial_j \phi)^2 + \\ (\frac{1}{8}\dot{\alpha}^2 - \frac{1}{2}\mu^2)\phi^2 + \frac{1}{2}exp(-\alpha)\lambda\phi^4] = 0 \end{cases} \tag{20}$$

That is equivalent to

$$\begin{cases} \frac{\partial H(\alpha)}{\partial \phi} \sim [-\frac{1}{2}\dot{\alpha}\partial_0 \phi + (\frac{1}{4}\dot{\alpha}^2 - \mu^2)\phi + exp(-\alpha)\lambda\phi^3] = 0 \\ \frac{\partial H(\alpha)}{\partial \alpha} \sim [-\frac{1}{2}\dot{\alpha}\phi\partial_0 \phi + \frac{1}{2}\sum_{j=0}^{3}(\partial_j \phi)^2 \\ +(\frac{1}{8}\dot{\alpha}^2 - \frac{1}{2}\mu^2)\phi^2 + \frac{1}{2}exp(-\alpha)\lambda\phi^4] = 0 \end{cases} \tag{21}$$

Now consider different solutions for (20) or (21).
I.In case of deformation parameter α weakly dependent on time

$$\dot{\alpha} \approx 0. \tag{22}$$

Most often this happens at low energies (far from the Plancks).
Actually, as $\alpha = l_{min}^2/a(t)^2$, where $a(t)$ is the measuring scale, (22) is nothing else but $\alpha \approx 0$, and the process takes place at low energies or at rather high energies but at the same energy scale, meaning that scale factor $a(t)^{-2}$ is weakly dependent on time. The first case is no doubt more real. In both cases, however, we have a symmetry breakdown and for minimum of $\tilde{\sigma}$ in case under consideration

$$\tilde{\sigma} = \pm\mu\lambda^{-1/2}exp(\alpha/2). \tag{23}$$

Based on the results of [1]– [3], it follows that

$$< 0 \mid \phi \mid 0 >_\alpha = exp(-\alpha) < 0 \mid \phi \mid 0 >, \tag{24}$$

and we directly obtain

$$< 0 \mid \tilde{\sigma} \mid 0 >_\alpha = \pm\mu\lambda^{-1/2}exp(-\alpha/2). \tag{25}$$

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Then in accordance with the exact formula [22], [23] we shift the field $\phi(\alpha)$

$$\phi(\alpha) \Rightarrow \phi(\alpha) + <0 \mid \tilde{\sigma} \mid 0>_\alpha = exp(-\alpha/2)(\phi + \sigma) = \phi(\alpha) + \sigma(\alpha), \tag{26}$$

where $\sigma = \pm\mu\lambda^{-1/2}$- minimum in a conventional nondeformed case [22]. When considering α - deformed Lagrangian (16) for the shifted field $\phi + \sigma$

$$L(\alpha) = L(\alpha, \phi + \sigma), \tag{27}$$

we obtain as expected a massive particle the squared mass of that contains, as compared to a well-known case, the multiplicative exponential supplement $exp(-\alpha)$:

$$m_\phi^2 = 2\mu^2 exp(-\alpha), \tag{28}$$

leading to the familiar result for low energies or correcting the particles mass in the direction of decreasing values for high energies.

II. Case of $\dot{\alpha} \neq 0$.

This case is associated with a change from low to higher energies or vice versa. This case necessitates the following additional assumption:

$$\partial_0\phi \approx 0. \tag{29}$$

What is the actual meaning of the assumption in (29)? It is quite understandable that in this case in α - deformed field $\phi(\alpha) = exp(-\frac{\alpha(t)}{2})\phi$ the principal dependence on time t is absorbed by exponential factor $exp(-\frac{\alpha(t)}{2})$. This is quite natural at sufficiently rapid changes of the scale (conforming to the energy) that is just the case in situation under study.

Note that in the process a change from Lagrangian to the Hamiltonian formalism (17) for α - deformed Lagrangian $L(\alpha)$ (16) holds true since, despite the condition of (29), we come to

$$\partial_0\phi(\alpha) \neq 0 \tag{30}$$

due to the presence of exponential factor $exp(-\frac{\alpha(t)}{2})$.

Proceed to the solution for (20) (and respectively (21)). Taking (29) into consideration, for a minimum of $\tilde{\sigma}$ in this case we obtain

$$\tilde{\sigma} = \pm(\mu^2 - \frac{1}{4}\dot{\alpha}^2)^{1/2}\lambda^{-1/2}exp(\alpha/2). \tag{31}$$

So, the requisite for the derivation of this minimum will be as follows:

$$4\mu^2 - \dot{\alpha}^2 \geq 0 \tag{32}$$

or with an assumption of $\mu > 0$

$$-2\mu \leq \dot{\alpha} \leq 2\mu. \tag{33}$$

Assuming that $\alpha(t)$ is increasing in time, i.e. on going from low to higher energies and with $\dot{\alpha} > 0$, we have

$$0 < \dot{\alpha} \leq 2\mu \tag{34}$$

or

$$\alpha(t) \leq 2\mu t. \tag{35}$$

From where it follows that on going from low to higher energies, i.e. with increasing energy of model(16), two different cases should be considered.

IIa. Symmetry breakdown when $\alpha(t) < 2\mu t$

$$\tilde{\sigma} = \pm(\mu^2 - \frac{1}{4}\dot{\alpha}^2)^{1/2}\lambda^{-1/2}exp(\alpha/2).$$

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IIb. Symmetry restoration when $\alpha(t) = 2\mu t$ as in this case

$$\tilde{\sigma} = 0.$$

Because of $\alpha(t) \sim a(t)^{-2}$, these two cases IIa and IIb may be interpreted as follows: provided $a(t)$ increases more rapidly than $(2\mu t)^{-1/2}$ (to within a familiar factor), we have a symmetry breakdown at hand as in the conventional case (14), whereas for similar increase a symmetry restoration occurs. This means that at sufficiently high energies associated with scale factor $a(t) \sim (2\mu t)^{-1/2}$ the broken symmetry is restored.

Provided that in this case the time dependence of $a(t)$ is exactly known, then by setting the equality

$$a(t) = l_{min}(2\mu t)^{-1/2}$$

and solving the above equation we can find t_c - critical time for the symmetry restoration. Then the critical scale (critical energies)

$$a(t_c) = l_{min}(2\mu t_c)^{-1/2},$$

actually the energies whose symmetry is restored, and finally the corresponding critical point of the deformation parameter

$$\alpha(t_c) = l_{min}^2 a(t_c)^{-2} = 2\mu t_c.$$

This point is critical in a sense that for all points with the deformation parameter below its critical value

$$\alpha(t) < \alpha(t_c)$$

the symmetry breakdown will be observed. Obviously, in case under study the energy is constantly growing with corresponding lowering of the scale and hence

$$a(t) \sim t^\xi, \xi < 0.$$

Of particular interest is a change from high to lower energies. This is associated with the fact that all cosmological models may be involved, i.e. all the cases where scale $a(t)$ is increased due to the Big Bang [20], [21].

For such a change with the assumption that $\alpha(t)$ diminishes in time and $\dot{\alpha} < 0$ we obtain

$$-2\mu \leq \dot{\alpha} < 0. \tag{36}$$

Since by definition $\alpha(t) > 0$ is always the case and considering $\alpha(t)$ as a negative increment (i.e. $d\alpha(t) < 0$), we come to the conclusion that the case under study is symmetric to IIa and IIb.

IIc. For fairly high energies, i.e. for $\alpha(t) = 2\mu t$ or scale $a(t)$ that equals to $(2\mu t)^{-1/2}$(again to within the familiar factor), there is no symmetry breakdown in accordance with case IIb.

IId. On going to lower energies associated with $\alpha(t) < 2\mu t$ there is a symmetry breakdown in accordance with case IIa and with the formula of(31). Note that in this case energy is lowered in time and hence the scale is growing, respectively. Because of this,

$$a(t) \sim t^\xi, \xi > 0.$$

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For the specific cases with exactly known relation between $a(t)$ and t one can determine the points without the symmetry breakdown. In cosmology [21] in particular we have

1) in a Universe dominated by nonrelativistic matter

$$a(t) \propto t^{2/3},$$

i.e.

$$a(t) \approx a_1 t^{2/3}.$$

From whence at the point of unbroken symmetry

$$a_1 t^{2/3} \approx (2\mu t)^{-1/2}, \tag{37}$$

directly giving the critical time t_c

$$t_c \approx (2\mu)^{-3/7} a_1^{-6/7}, \tag{38}$$

and for $t > t_c$ the symmetry breakdown is observed (case IId).
In much the same manner

2) in the Universe dominated by radiation

$$a(t) \propto t^{1/2},$$

i.e.

$$a(t) \approx a_2 t^{1/2}$$

from where for t_c we have

$$t_c \approx (2\mu)^{-1/2} a_2^{-1}, \tag{39}$$

and again for $t > t_c$ the symmetry is broken.
Here it is interesting to note that despite the apparent symmetry of cases IIa,IIb and IId , IIc, there is one important distinction.
In cases IId and IIc time t is usually (in cosmological models as well [20], [21]) counted from the Big Bang moment and therefore fits well to the associated time (temperature) coordinate of the lattice Lat^{τ}_{α} (section 2 and [1], [6]. As a result, when the critical time t_c is known, one can find the critical point of the above-mentioned lattice as follows: $(\alpha_c, \tau_c) = (\widetilde{\alpha_c}, \tau_c)$, where all the three coordinates of the space part Lat^{τ}_{α}, i.e. in $(\widetilde{\alpha_c})$, are equal to

$$\alpha_c = l^2_{min} a(t_c)^{-2},$$

and

$$\tau_c = T^2_c / T^2_{max},$$

where $T_c \sim 1/t_c$ ((8) in section 2 and [6], [19]).

Thus, in these cases all points $Lat^{\tau}_{\underset{\sim}{\alpha}}$, for which the following conditions are satisfied:

$$\begin{cases} \alpha < \alpha_c, \\ \tau < \tau_c \end{cases} \tag{40}$$

are associated with a symmetry breakdown, whereas at the critical point (α_c, τ_c) no symmetry breakdown occurs. As seen from all the above formulae, the point of the retained symmetry is associated with higher temperatures and energies than those (40), where a symmetry breakdown takes place, in qualitative agreement with the principal results of [22].

Note that for cases IIa and IIb time t is a certain local time of the quantum process having no direct relation to the time (temperature) variable of lattice $Lat^{\tau}_{\underset{\sim}{\alpha}}$. Because of this, it is possible to consider only the critical value at the space part $Lat_{\underset{\sim}{\alpha}}$ [1] of lattice $Lat^{\tau}_{\underset{\sim}{\alpha}}$, i.e. $\alpha_c \in Lat_{\underset{\sim}{\alpha}}$, all other inferences remaining true with a change of (40) by

$$\alpha < \alpha_c. \tag{41}$$

However, provided there is some way to find temperature T_c that is associated with a symmetry restoration(e.g, [22] case IIb), one can directly calculate τ_c and finally change (41)by (40).

In any case conditions $0 < \alpha \leq 1/4; 0 < \tau \leq 1/4$ (see section 2 and [1]– [6]) impose constraints on the model parameters (40) and hence on μ, which may be found in the explicit form by solving the following inequality:

$$0 < \alpha_c \leq 1/4.$$

IV. CONCLUSION

Thus, from primary analysis of such a simple model as (14) it is seen that its α - deformation (16) contributes considerably to widening the scope of possibilities for a symmetry breakdown and restoration.

1) At low energies (far from the Plancks) it reproduces with a high accuracy (up to $exp(-\alpha) \approx 1$) the results analogous to those given by a nondeformed theory [22].

2) On going from low to higher energies or vice versa it provides new cases of a symmetry breakdown or restoration depending on the variation rate of the deformation parameter in time, being capable to point to the critical points of the earlier considered lattice $Lat^{\tau}_{\underset{\sim}{\alpha}}$, i.e. points of the symmetry restoration.

3) This model makes it possible to find important constraints on the parameters of the initial model having an explicit physical meaning.

It should be noted that for α - deformed theory (16) there are two reasons to be finite in ultraviolet:

- cut-off for a maximum momentum p_{max};

- damping factors of the form $exp(-\alpha)$, where in the momentum representation $\alpha = p^2/p^2_{max}$ in each order of a perturbation theory suppressing the greatest momenta.

As a result, some problems associated with a conventional theory [22] (divergence and so on) in this case are nonexistent.

In his further works the author is planning to proceed to α - deformations (symmetry breakdown and restoration, critical points an so forth) of more elaborate theories involving the gauge A_μ and spinor ψ fields.

[1] A.E. Shalyt-Margolin, The Universe as a Nonuniform Lattice in Finite-Volume Hypercube I. Fundamental Definitions and Particular Features, Intern. Journ. of Mod.Phys.D.13 (2004)853;

[2] A.E.Shalyt-Margolin, Fundamental Length,Deformed Density Matrix and New View on the Black Hole Information Paradox,[gr-qc/0207074]

[3] A.E.Shalyt-Margolin and J.G.Suarez,Quantum Mechanics at Planck's scale and Density Matrix, Intern.Journ.of Mod.Phys. D.12(2003)1265;

[4] A.E.Shalyt-Margolin,Non-Unitary and Unitary Transitions in Generalized Quantum Mechanics, New Small Parameter and Information Problem Solving, Mod. Phys. Lett. A, Vol 19, No. 5 (2004)391;

[5] A.E.Shalyt-Margolin,Pure States, Mixed States and Hawking Problem in Generalized Quantum Mechanics, Mod.Phys.Lett.A19(2004)2037

[6] A.E.Shalyt-Margolin,A.Ya.Tregubovich,Deformed Density Matrix and Generalized Uncertainty Relation in Thermodynamics, Mod. Phys. Lett. A, Vol.19, No.1(2004)71.

[7] D.V.Ahluwalia,Wave-Particle duality at the Planck scale: Freezing of neutrino oscillations, Phys.Lett. A275 (2000)31;Interface of Gravitational and Quantum Realms, Mod.Phys.Lett. A17(2002)1135

[8] G.Veneziano,A stringly nature needs just two constant *Europhys.Lett.* 2(1986)199;D.Amati,M.Ciafaloni and G.Veneziano,Can spacetime be probed below the string size? *Phys.Lett.* B216(1989)41; E.Witten, Reflections on the Fate of Spacetime *Phys.Today*,49(1996)24

[9] M.Maggiore,Quantum Groups,Gravity and Generalized Uncertainty Principle, Phys.Rev.D49(1994)5182

[10] D. V. Ahluwalia,Quantum Measurement, Gravitation, and Locality, Phys.Lett. B339 (1994)301

[11] L.Garay,Quantum Gravity and Minimum Length *Int.J.Mod.Phys.A.* 10(1995)145[gr-qc/9403008]

[12] R.J.Adler and D.I.Santiago. On Gravity and the Uncertainty Principle. Mod.Phys.Lett. **A14**, 1371-1377 (1999).

[13] S.Capozziello,G.Lambiase and G.Scarpetta, The Generalized Uncertainty Principle from Quantum Geometry, Int.J.Theor.Phys. 39 (2000),15

[14] L.Faddeev. Mathematical View on Evolution of Physics. Priroda. **5**, 11-18, (1989).

[15] W.Heisenberg. Uber den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik. Zeitsch.fur Phys. **43**, 172-184, (1927).

[16] M.Maggiore, The algebraic structure of the generalized uncertainty principle,Phys.Lett.B319(1993)83

[17] A.Kempf, G.Mangano, R.B.Mann. Hilbert Space Representation of the Minimal Length Uncertainty Relation. Phys.Rev. D52 (1995)1108.

[18] R.P.Feynman, Statistical Mechanics,A Set of Lectures ,California, Institute of Technology.W.A.Benjamin,Inc.Advanced Book Program Reading,Massachusets 1972

[19] A.E.Shalyt-Margolin and A.Ya.Tregubovich, Generalized Uncertainty Relations in a Quantum Theory and Thermodynamics From the Uniform Point of View [gr-qc/0204078]

[20] A.Linde,Particle physics and Inflationary Cosmology,(Harwood,Chur,1990).

[21] D. Langlois,Inflation, quantum fluctuations and cosmological perturbations,Lectures delivered at the Cargese School of Physics and Cosmology, Cargese, France, August 2003, [hep-th/0405053]

[22] A.Linde,Phase transitions in gauge theories and cosmology, Rep.Prog.Phys.,42 (1979) 389

[23] M.Peskin, D.Schroeder, The Introduction to Quantum Field Theory, Addison-Wesley Publishing Company