

## ON PROBLEM DEFINITION FOR PARTIAL DIFFERENTIAL EQUATIONS IN CASE OF CLASSICAL AND NUMERICAL SOLUTIONS

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Classical solutions for various boundary value problems and problems with integral conditions for partial differential equations were tackled by authors ([1–3] and other). We present analytical solutions in case of two independent variables for the following problems:

- Cauchy problem for hyperbolic equation with constant coefficients. Its operator is the composition of first-order operators;
- mixed boundary value problems for a second-order hyperbolic equation;
- mixed boundary value problems for a bi-wave equation;
- mixed boundary value problems for a wave equation with integral boundary conditions;
- a wave equation with integral conditions in the domain of the solution;
- control problems of Cauchy or boundary value conditions when values of the unknown function or its derivatives are specified inside the domain of the solution.

We show that classical solutions defined in the whole domain of the equation depend on homogeneous compatibility conditions for given boundary conditions at common angular points of the boundary. These compatibility conditions are necessary and sufficient. When these conditions become inhomogeneous, well-posed problems need to be defined in a different way. For example, they can be posed as conjugation problems with conjugation conditions on proper characteristics inside the domain of the unknown function.

On the other hand, there are well-known numerical methods for solving boundary value problem and other kinds of problems, which assume the existence of classical solutions. Generally, the literature on numerical methods does not discuss or does not stress the importance of compatibility conditions for given boundary conditions. However, this is crucial for full and correct definition of various problems under discussion and for the correct usage of corresponding numerical methods.

### REFERENCES

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