SIMULATION OF ADAPTIVE CONTROL SYSTEM WITH CONFLICT FLOWS OF NON-HOMOGENEOUS REQUESTS

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Abstract

An adaptive control system with conflict flows of non-homogeneous requests is considered in the paper. A mathematical model of the system is a vector Markov sequence with a countable state space. Components of the Markov sequence satisfy certain functional recurrence relations. The main result of the work is a numerical research of the system by simulation. In particular, some sample estimates for the mean sojourn time of a single request from different queues are presented.
Keywords: data science, adaptive control system, conflict flow, non-homogeneity

1 Introduction

The adaptive non-cyclic control system with two conflict flows of requests is investigated here using computer-aided simulation. The algorithm controls the input flows using information about queues lengths and the order of requests arrivals. Conflictness of flows means here existence impossibility for the time intervals when requests from different flows are serviced simultaneously. Each flow here consists of requests of different types. In [1, 2], the authors showed the input flows can be approximated by non-ordinary Poisson flows. Thus, two statistically independent flows Π_1 and Π_2 are serviced. Request arrival moments in the flow Π_j occur with intensity of λ_j (for $j = 1, 2$), and a group with k requests arrives with probability $Q_j(k)$ where

$$Q_j(1) = \Big(1 + \alpha_j + \frac{\alpha_j\beta_j}{1-\gamma_j}\Big)^{-1} = p_j, \quad Q_j(2) = \alpha_j\Big(1 + \alpha_j + \frac{\alpha_j\beta_j}{1-\gamma_j}\Big)^{-1}$$

$$Q_j(k) = \alpha_j\beta_j\gamma_j^{k-3}\Big(1 + \alpha_j + \frac{\alpha_j\beta_j}{1-\gamma_j}\Big)^{-1}, \quad k \geq 3,$$

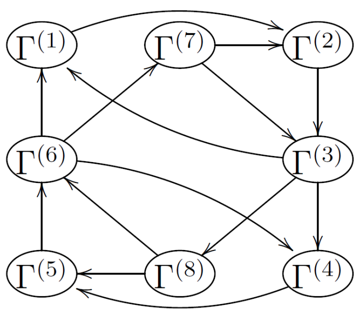
α_j, β_j and γ_j are some parameters that have a certain physical meaning [1]. Let the random variable $\eta_j(t)$ determine the number of requests received by the flow Π_j during the time interval $[0, t)$. Denote the probability $\mathbf{P}(\eta_j(t) = k)$ by the function $P_j(t, k)$.

In our previous work the following equality was obtained:

$$P_j(t,k) = e^{-\lambda_j t} \sum_{n=0}^{[\frac{k}{2}]} \alpha_j^n \frac{(\lambda_j t p_j)^{k-n}}{n!(k-2n)!} +$$

$$+ e^{-\lambda_j t} \sum_{n=0}^{[\frac{k}{2}]} \alpha_j^n \sum_{m=1}^{\min\{k-2n,n\}} \beta_j^m \sum_{l=0}^{k-2n-m} \gamma_j^l \frac{(\lambda_j t p_j)^{k-n-m-l} C_{m+l-1}^l}{(n-m)!m!(k-2n-m-l)!}, \quad k \geq 0.$$

The queue for each of the flows is assumed unlimited. The queueing system is assumed lossless. The server state space is $\Gamma = \{\Gamma^{(1)}, \Gamma^{(2)}, \ldots, \Gamma^{(8)}\}$. The following graph specifies transitions of server states



The state $\Gamma^{(3j-2)}$ corresponds to the first stage of the service period for the j-th flow. The service duration for one request from queue O_j (i.e. from the flow Π_j) is a constant value $\mu_{j,1}^{-1}$. Let T_{3j-2} be the duration of the state $\Gamma^{(3j-2)}$. The state $\Gamma^{(3j-1)}$ corresponds to the second stage of the service period for the j-th flow. The service duration of one request in this state is the constant value $\mu_{j,2}^{-1} < \mu_{j,1}^{-1}$. The duration of the state $\Gamma^{(3j-1)}$ is a random variable taking on the values kT_{3j-1}, $k = \overline{1, n_j}$, where n_j is the given maximum number of prolongations. The parameter K_j is the queue size, above which there is the prolongation. The state $\Gamma^{(3j)}$ corresponds to the server setup after servicing the j-th flow. The duration of the state is T_{3j}. The service duration of one request in the state $\Gamma^{(3j)}$ is $\mu_{j,2}^{-1}$. The state $\Gamma^{(6+j)}$ corresponds to the first stage of the service period for the j-th flow in the case when an instantaneous transition to the state $\Gamma^{(3j)}$ is possible. The duration of the state $\Gamma^{(6+j)}$ is a random variable. Its largest value is T_{3j-2}. The constant values T_k, $k = \overline{1,6}$, are defined by

$$T_{3j-2} = \mu_{j,1}^{-1} + l_{3j-2}\theta_j \mu_{j,1}^{-1}, \quad T_{3j-1} = l_{3j-1}\theta_j \mu_{j,2}^{-1}, \quad T_{3j} = l_{3j}\theta_j \mu_{j,2}^{-1}, \tag{1}$$

where $l_{3j-2} \in \{0, 1, 2, \ldots\} = X$, l_{3j-1}, $l_{3j} \in \{1, 2, \ldots\}$ and θ_j are parameters. The value θ_j, $0 < \theta_j \leq 1$, denotes the portion of the service time which needs pass before the next request can begin its servicing. In case $\theta_j < 1$, several requests can be serviced simultaneously. The ratio (1) means that the server changes its state when the service of some request is finished. The maximum possible number of served requests is $1 + l_{3j-2}$ in the state $\Gamma^{(3j-2)}$, one is kl_{3j-1} for the state $\Gamma^{(3j-1)}$ and one is the integer part of the number $1/\theta_j$ for the state $\Gamma^{(3j)}$.

2 Construction of the mathematical model

We observe the system at random time instants τ_i, $i = 0, 1, \dots$, or on intervals $[\tau_i, \tau_{i+1})$, $i \geq 0$. Here, the value τ_0 is the initial moment of time, and τ_i, $i > 0$, are the moments of server state change. Set $y_0 = (0, 0)$, $y_1 = (1, 0)$, $y_2 = (0, 1)$. For $i \geq 0$ and $j = 1, 2$, we define the following random variables and elements:

1. $\Gamma_i \in \Gamma$ — the server state during the interval $[\tau_i, \tau_{i+1})$;

2. $\eta_{j,i} \in X$ is the number of flow Π_j requests that enter the system during the interval $[\tau_i, \tau_{i+1})$, and $\eta_i = (\eta_{1,i}, \eta_{2,i})$;

3. η_i' is a random vector. The vector η_i' takes on the value y_0 if no requests have entered the system during the interval $[\tau_i, \tau_{i+1})$, otherwise the value y_j if the request (or requests) of the flow Π_j is the first during the i-th interval;

4. $\kappa_{j,i} \in X$ is the number of requests for the flow Π_j in the system at time τ_i, and $\kappa_i = (\kappa_{1,i}, \kappa_{2,i})$;

5. $\xi_{j,i}$ is the maximum possible number of flow Π_j requests that the system can service during the interval $[\tau_i, \tau_{i+1})$, and $\xi_i = (\xi_{1,j}, \xi_{2,j})$.

An adaptive algorithm for conflict flow control is defined by a function $u(\cdot, \cdot, \cdot)\colon \Gamma \times$ $\times X^2 \times \{y_0, y_1, y_2\} \to \Gamma$ by virtue of the following recurrence relations

$$\Gamma_{i+1} = u(\Gamma_i, \kappa_i, \eta_i') = \\ = \begin{cases} \Gamma^{(3j-2)}, & \left\{\left[\Gamma_i = \Gamma^{(3s)}\right] \& \left[(\kappa_{j,i} > 0) \vee (\kappa_{s,i} \geq K_s) \vee (\eta_i' = y_j)\right]\right\} \vee \\ & \vee \left\{\left[\Gamma_i = \Gamma^{(3j)}\right] \& [\kappa_{s,i} = 0] \& [\kappa_{j,i} \leq K_j] \& [\eta_i' = y_j]\right\}, \\ \Gamma^{(3j-1)}, & \left\{\Gamma_i = \Gamma^{(3j-2)}\right\} \vee \left\{\left[\Gamma_i = \Gamma^{(6+j)}\right] \& [\eta_i' = y_j]\right\}, \\ \Gamma^{(3j)}, & \left\{\Gamma_i = \Gamma^{(3j-1)}\right\} \vee \left\{\left[\Gamma_i = \Gamma^{(6+j)}\right] \& [\eta_i' \neq y_j]\right\}, \\ \Gamma^{(6+j)}, & \left[\Gamma_i = \Gamma^{(3s)}\right] \& [\kappa_{j,i} = 0] \& [\kappa_{s,i} < K_s] \& [\eta_i' = y_0], \end{cases} \tag{2}$$

hereinafter in the work $j, s = 1, 2$, $j \neq s$, $i \geq 0$. The queue length dynamics is determined by functions $v_j(\cdot, \cdot, \cdot, \cdot)\colon \Gamma \times X^2 \times X^2 \times X^2 \to X$ and the following recurrence relations

$$\kappa_{j,i+1} = v_j(\Gamma_i, \kappa_i, \eta_i, \xi_i) = \begin{cases} \max\{0, \kappa_{j,i} + \eta_{j,i} - \xi_{j,i}\} & \text{if } \Gamma_i \in \Gamma \backslash \{\Gamma^{(3)}, \Gamma^{(6)}\}; \\ \eta_{j,i} + \max\{0, \kappa_{j,i} - \xi_{j,i}\} & \text{if } \Gamma_i \in \{\Gamma^{(3)}, \Gamma^{(6)}\}. \end{cases} \tag{3}$$

Relations (2) and (3) allow to us to study the vector sequence $\{(\Gamma_i, \kappa_i); i = 0, 1, \dots\}$. The sequence is a probabilistic model of the queueing system for adaptive control of conflict flows and for service of non-homogeneous requests. The properties of the vector Markov sequence were investigated in [3, 4, 5]. In particular, the conditions for the stationary probability distribution existence were obtained.

Unfortunately, it is not possible to derive analytically the important performance characteristics of the system under study. Therefore, a computer-aided simulation model is built to determine some important characteristics of the system. Simulation results can be interpreted in terms of a transport intersection operation.

3 System simulation model

The simulation takes places in the discrete time-scale $\{\tau_i; i = 0, 1, \dots\}$ and a realization of the sequence $\{(\Gamma_i, \kappa_i); i = 0, 1, \dots\}$ is generated together with all random objects involved in equations (2) and (3). Besides that, arrival times are stored for all requests, it allow to keep track of sojourn times of every request in the system. Denote by ν_j the sample estimate for the mean sojourn time of requests from the flow Π_j. The sample estimate of the sojourn time of an arbitrary request is given by $\nu = \frac{\lambda_1 M_1 \nu_1 + \lambda_2 M_2 \nu_2}{\lambda_1 M_1 + \lambda_2 M_2}$. Here M_1 and M_2 are the mathematical expectations of the number of requests in a group and $M_j = (1 + 2\alpha_j + \alpha_j\beta_j(2/(1-\gamma_j) + 1/(1-\gamma_j)^2))p_j$. The simulation model is implemented as a program written in C++.

As an example, we present the computational results concerning the estimate of the mean sojourn time of an arbitrary request in the system with the following parameters: $\lambda_1 = 0.4$, $\lambda_2 = 0.3$, $\alpha_1 = 1.1$, $\beta_1 = 0.1$, $\gamma_1 = 0.01$, $\alpha_2 = 1.1$, $\beta_2 = 0.1$, $\gamma_2 = 0.01$. The parameters of the adaptive algorithm are $T_1 = T_4 = 5$, $T_2 = T_5 = 1$, $T_3 = T_6 = 2$, $n_1 = n_2 = 7$, $K_1 = K_2 = 10$, $\theta_1 = \theta_2 = 1$, $\mu_{1,1} = \mu_{2,1} = 2/3$, $\mu_{1,2} = \mu_{2,2} = 1$. The parameters $T_1, \dots, T_6$ are given in seconds. The parameters λ_1, λ_2, $\mu_{1,1}$, $\mu_{2,1}$, $\mu_{1,2}$, $\mu_{2,2}$ have the measurement units of $(\text{seconds})^{-1}$. Other parameters are dimensionless. With these parameters of adaptive flow control the sample estimate of the mean sojourn time is 13.55 seconds.

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