introduction of the polynomial extended root locus notion, which allows to obtain a descriptive picture of the polynomial roots dynamics under coefficient variations and to disclose on this basis the cause of instability. The intervals of uncertainty for each coefficient being set up are specified along the root locus branches. The nearest stable polynomial to the given unstable one in terms of a distance along the root locus branches is being found. Solving the task of ensuring the required quality (e.g. the polynomial stability margin) could be one of the possible directions for further development of the method.

The developed method is new and allows to extend the application sphere of the root locus method, which is traditionally considered to be the method of systems synthesis by only a single parameter (coefficient) variation and with only one variable parameter (coefficient), in both directions, systems synthesis by many parameters variation and systems synthesis with many parameters variation.

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ROBUST SYNTHESIS METHOD FOR LINEAR CASCADED SYSTEM WITH LOW ORDER CONTROLLER

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The controllers synthesis with determined structure, for instance, proportional-integral differential (PID), is an important problem for the systems with high order plant, because these controllers are typically employed in practical industrial applications [1].

The time scale methods developed by L. Anderson [2], P. V. Kokotovich and coauthors [3] is appropriate for the cascaded synthesis, based on plant model reduction. It is related with the modal control synthesis problem. So, the polynomial roots placement on the complex plane can offer the time scale for the external and internal control loops of the cascaded system. These techniques can be also extended for robust synthesis of the systems with interval plants in order to simplify the roots deviation analysis, caused by plant parameters variation. Consider the linear cascade system described as follow

$$\dot{x}_R = A_R x_R + B_R(y^* - y), \tag{1}$$

$$u = C_R x_R + D_R(y^* - y), \tag{2}$$

$$\dot{x}_P = A_P x_P + B_P u, \tag{3}$$

$$y = C_P x_P, \tag{4}$$

where x_R is an n-vector states of the regulators (controller), u is a scalar output of the controller, x_p is an n-vector states of the plant, $y*$, y are the scalar input and the scalar output of the system respectively. The system (1)-(4) contains the matrixes A_R, B_R, C_R, D_R of the controller and matrixes A_P, B_P, C_P of the plant. If design $x^T = (x_R, x_P)$, and

$$A = \begin{bmatrix} A_R & -B_R C_P \\ B_P C_R & A_{PC} \end{bmatrix}, \quad B = \begin{bmatrix} B_R \\ B_P D_R \end{bmatrix},$$

where $A_{PC} = A_P - B_P D_R C_P$, the system (1)-(4) can be rewritten as follow:

$$\dot{x} = Ax + By^*, \quad y = Cx.$$

For the system with two loops the matrix A obtains the form

$$A = \begin{bmatrix} A_{R1} & 0 & 0 & -B_{R1}C_{P1} \\ B_{R0}C_{R1} & A_{R0} & -B_{R0}C_{P0} & -B_{R0}D_{R1}C_{P1} \\ B_{P0}D_{R0}C_{R1} & B_{P0}C_{R0} & A_{PC00} & A_{PC01} \\ 0 & 0 & A_{PC10} & A_{PC11} \end{bmatrix}.$$

In this expression the internal subsystem is indexed by 0 and has a matrix

$$A_0 = \begin{bmatrix} A_{R0} & -B_{R0}C_{P0} \\ B_{P0}C_{R0} & A_{PC0} \end{bmatrix}.$$

As usual, the internal loop of the cascaded system provides the faste mode, and the external one provides a slow mode of dynamics. We assume that a little positive value ε is the relative value of the slow mode dominant eigenvalue $|s_{di}|$. So $\varepsilon = s_{di}/s_{d0}$, where s_{d0} is the fast mode of the system, and let $0 < \varepsilon < 0.5$. Then, the matrix may be rewritten as follow

$$A_0 = \begin{bmatrix} A_{R0} & -B_{R0}C_{P0} \\ B_{P0}C_{R0} & A_{PC0} \end{bmatrix} = \frac{1}{\varepsilon}\begin{bmatrix} \epsilon A_{R0} & -\varepsilon B_{R0}C_{P0} \\ B_{P0}\bar{C}_{R0} & \varepsilon A_{PC00} \end{bmatrix} = \frac{1}{\varepsilon}\bar{A}_0.$$

Hence, the equations of the internal loop with assumption became algebric, and the reduced system model is as follow $\dot{\bar{x}}_1 = \bar{A}_1\bar{x}_1 + \bar{B}_1 y^*$.

This model describes the slow mode, controlled by R1 and presents the main component of system dynamics. The polynomial of the closed loop external subsystem obtains the simplified low order form. Now, it is easy to establish the dependence between polynomial coefficients and polynomial roots. The polynomial is useful for outer loop controller parameterization. After the external loop controller parameterization the inner loop synthesis is available with respect to the denoted fast mode s_{d0}. The relative difference σ beetwen reduced and full order polynomial roots location depends on the value of ε as follow [4] $\sigma = \frac{|\Delta s_i|}{|\bar{s}_i|} \leq m\varepsilon$, where m is the inner loop polynomial order. So, the roots placement error in the system relatively to the reduced model can be decreased depending on the choise of the ε value. The synthesed system polynomial roots must be verificated. The verification can be get up by the computation of these roots for full (not reduced) plant model.

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