Nonlinear Phenomena in Complex Systems, vol. **19**, no. 4 (2016), pp. 368 - 377

Scale Invariance of News Flow Intensity Time Series

S. P. Sidorov,* A. R. Faizliev, and V. A. Balash
Saratov State University, 83 Astrakhanskaya Str, 410012 Saratov, RUSSIA
(Received 6 October, 2016)

In this paper we examine the presence of self-similarity in flow intensity of economic and financial news taken from a nine-month period of 2015. Since there is a close relationship between long range dependent and self-similar processes, we use two methods – the detrended fluctuation analysis (DFA) and the averaged wavelet coefficient (AWC) method – to estimate both the long range correlation and the self-similarity exponent (the Hurst exponent), respectively. Empirical results obtained by this methods show that time series of news intensity exhibit self-similarity (as well as a long memory property). The Hurst exponent (as well as the long-range correlation exponent) is greater than 0.5 over three orders of magnitude in time ranging from several minutes to dozen of days. Estimates of the Hurst exponent obtained by AWC are very close to the estimates of the long-range correlation exponent obtained by DFA for almost all analyzed time series. By using decouple scales and multi-scale approaches, the DFA and the AWC methods allowed us to reveal a strong scaling behavior as well as to detect a distinct crossover effect.

PACS numbers: 05.40.-a, 05.45.Df, 05.45.Tp
Keywords: long-range correlation, detrended fluctuation analysis, time series, auto-correlation

1. Introduction

A wide collection of complex systems and processes (in biology, physics, engineering, human behavior) and their generated data signals have been seen to exhibit properties of scale invariance [1]. Scale invariance can be presented in different forms, among them are self-similarity and long-range dependence. Self-similarity occurs when two subsets of the whole set are invariant in statistical distribution on different scales. Long-range dependence means a power-law slowly decaying dependence of far past on future. Studying self-similarity and long-range dependence in these diverse processes provides insight into the dynamics of complex systems and enables to predict future outcomes based on current information.

It is said that a process $\{X(t)\}_{t\in\mathbb{R}}$ is self-similar (or statistically self-affine), if and only if for all $\beta > 0$ the process $\beta^{-H}X(\beta t)$ has the same second-order statistical properties as $X(t)$, H is called self-similarity parameter.

A second-order stationary stochastic process $\{X(t)\}_{t\in\mathbb{R}}$ with covariance function C_Y is said to be long-range dependant if

$$C_Y(s) \sim r_C|s|^{-\gamma},\ s \to \infty,\ \gamma \in (0,1).$$

It is well-known [1] that there is a close relationship between long range dependent and self-similar processes. In particular, any self-similar process with self-similarity parameter $0.5 < H < 1$ has long-range dependence and $\gamma = H$. On the other hand, some long range dependent processes are not self-similar (for example, FARIMA processes).

The first empirical research on self-similarity was published in 1951 by Hurst [2]. Over the last 50 years many methods and techniques have been developed to study scaling properties and the long-range dependence of time series exhibiting self-similar behavior. Various methods like rescaled range analysis [2], structure function method [3], the wavelet transform module maxima approach [4], the detrended fluctuation analysis [5], the detrending moving average analysis (DMA) [6], average wavelet coefficient method [7], among many others, have been

*E-mail: `sidorovsp@info.sgu.ru`

proposed for the characterization of self-similarity and long-range dependence.

Following the paper [8], in this work we analyze time series of news flow intensity and we show that such time series exhibit the self-similarity property. In contrast to the work [8], in this paper

1. we suggest using another technique – a wavelet based analyzing technique (Averaged Wavelet Coefficient (AWC) method) – to quantify the self-similarity property of news intensity time series;

2. we analyze a much bigger data period (189 instead of 43 trading days) and time series based on relatively recent news analytics data (from January 1, 2015 to September 22, 2015).

In this paper we will use two estimators – detrended fluctuation analysis of orders 1 (DFA) and AWC method – to examine long-range auto-correlation and self-similarity of time series of news flow intensity. We suppose that the usage of the two estimators may prevent us from obtaining one-sided results. By using a bigger data period, we could study the dependence of the Hurst exponent not only on intra day intervals, but at intervals of a few days duration.

2. Detrended fluctuation analysis

2.1. Types of auto-correlation

Let $X = (x_t)_{t=1}^{n}$ be a time series with large n and let $s \in \mathbb{N}$, $s \ll n$. Let $C(s)$ denote the (auto) correlation between $X_1 = (x_t)_{t=1}^{n-s}$ and $X_2 = (x_{t+s})_{t=1}^{n}$. The following types of correlation can be distinguished:

1. x_t are *uncorrelated*; it is clear that if X_1 and X_2 are uncorrelated then $C(s)$ must be equal to zero, $C(s) = 0$;

2. the case of *short-range correlations* of the $(x_t)_{t=1}^{n}$ leads to exponentially declining of $C(s)$, i.e. $C(s) \sim e^{-s/s_0}$, where s_0 is a decay time;

3. with long-range correlation of the $(x_t)_{t=1}^{n}$, $C(s)$ must follow a power-law dependence:

$$C(s) \sim s^{-\gamma}, \tag{1}$$

where $0 < \gamma < 1$.

Fourier transform, wavelet transform modulus maxima are classical methods for estimation of the correlation exponent γ, but they are often not appropriate due to noisy nature, non-stationarity and imperfect measurement of data x_t. That is why we use the following methods for estimation of the correlation.

2.2. DFA method

DFA was proposed in the papers [5], [9]. This method is used for studying the indirect scaling of the long-range auto-correlation in non-stationary time series. DFA method was effectively applied for solving many scientific and engineering problems, including analysis of DNA [10], [11], [12], biomedical signal processing [13], [14], [15], [16], [17], study of daily internet traffic dynamics [18], analysis of economical and finance time series [19], [20], [21], [22], [23], human gait behavior [24], [25]. However, DFA has some drawbacks [26]: DFA can lead to uncontrolled bias; DFA is more expensive than unbiased estimator; DFA cannot provide protection against nonlinear nonstationaries.

Nevertheless, the work [27] remarks that DMA and DFA remain 'the methods of choice' in determining the Hurst index of time series. DFA algorithm consists of five steps:

1. *Integration.* We calculate the cumulative deviate series as follows

$$y_k = \sum_{t=1}^{k} (x_t - \overline{x}), \ k = 1, 2, \ldots, n, \tag{2}$$

where $\overline{x} = \sum_{t=1}^{n} x_t$.

2. *Cutting.* We divide the $(y_k)_{k=1}^n$ into $n_s = [n/s]$ non-overlapping segments, each of length s, starting with y_1.

3. *Fitting.* For each segment $l = 1, \ldots, n_s$ we construct a fitting linear function P_l (trend) by means of least-square fit of the data $(y_k)_{k=(l-1)s+1}^{ls}$. Denote $y_k^* = P_l(k)$.

4. *Detrending.* We obtain the detranded time series

$$\epsilon_k(s) = y_k - y_k^* \qquad (3)$$

as the difference between the time series y_k and the the corresponding values of fitting linear regression P_l on the segment l.

5. For each of segments $l = 1, \ldots, n_s$ we find variance of residuals ϵ_i:

$$F_l^2(s) = \frac{1}{s}\sum_{i=1}^{s} \epsilon_{(l-1)s+i}^2(s). \qquad (4)$$

Then *DFA fluctuation function* is defined by

$$F(s) = F_{\mathrm{DFA}}^{[m]}(s) = \left(\frac{1}{n_s}\sum_{l=1}^{n_s} F_l^2(s)\right)^{1/2}, \qquad (5)$$

where m refers to the degree of fitting polynomials using on step 3.

Derivation of DFA can be found in the paper [28].

2.3. The auto-correlation parameter

The fluctuation functions $F(s)$ obtained in DFA allow us to examine the s-dependance of F. In the case of long-range power-law correlation of x_t, the F must follow a power-law

$$F(s) \sim s^{\alpha} \qquad (6)$$

for sufficiently large s. The fluctuation parameter α is related to the value of correlation exponent γ as follows (see [29])

$$\alpha = 1 - \gamma/2, \ 0 < \gamma < 1.$$

The correlation parameter α reflects the auto-correlation properties of time series in the following way:

1. $\alpha = 1/2$, the time series uncorrelated (white noise) or short-range correlated;

2. $\alpha < 1/2$, the time series anti-correlated;

3. $\alpha > 1/2$, the time series long-range power-law correlated;

4. $\alpha = 1$, pink noise ($1/f$ noise).

3. Wavelet-based methods

3.1. The Hurst exponent

The self-similarity parameter $0 < H < 1$ of self-affine processes is also called the Hurst (or roughness) exponent [3]. The Hurst exponent is commonly used for measuring the duration of long-range dependence of a stochastic process.

There are three possibilities:

- If $H = 0.5$ then $C(s) = 0$ which means that past and future increments are uncorrelated (Brownian motion);

- In case $H > 0.5$ we have $C(s) > 0$ and the increments are positively correlated (the process $\{X(t)\}_t$ is called persistent).

- If $H < 0.5$ then $C(s) < 0$ and increments are negatively correlated (the process $\{X(t)\}_t$ is called anti-persistent or anti-correlated).

3.2. The average wavelet coefficient method

The Average Wavelet Coefficient Method (AWC) was proposed in papers [30] and [7].

The method is used for measuring the temporal self-affine correlations of a time series by estimating its Hurst exponent. The AWC method is based on the wavelet transform (good review of the wavelet transform can be found in books [31] and [32]).

The strategy for the data-analysis by the AWC method consists of three main steps:

1. Wavelet transformation of the data $X(t)$ into the wavelet domain, $\mathcal{W}[X](a, b)$, where a, b are scale and location parameters, respectively [31], [32].

2. Then for a given scale a we can find a representative wavelet amplitude for that particular scale, and to study its scaling. To do so we calculate the averaged wavelet coefficient $W[X](a)$ according to the equation

$$W[X](a) = \langle|\mathcal{W}[X](a, b)|\rangle_b,$$

where $\langle\cdot\rangle_b$ denotes the standard arithmetic mean value operator with respect to the b.

3. For a self-affine process $X(t)$ with exponent H, the spectrum $W[X](a)$ should scale as $a^{H+0.5}$ [7]. Therefore, to find $H + 0.5$ we should plot $W[h](a)$ against scale a in a log-log plot. As it is pointed out in [7], a scaling regime consisting of a straight line in this plot implies a self-affine behavior of the data.

4. Empirical results

4.1. Time series of news flow intensity

Our data covers the period from January 1, 2015 to September 22, 2015 (i.e. 189 trading days). We consider all the news released during this period. Initially we carried out data selection and cleaning process. The criteria were the following:

1. We excluded all weekend news from our analysis as the average amount of a company week-end's news is much lower than the one of the weekdays. The same picture is true for all companies.

2. We considered only unique news. If a piece of news is related to a group of companies then the news will be multiplicated in the data for every company in the group (for example, if a news is related to a certain industry, providers of news analytics will repeat the news for every company in the industry). So we counted all multiplicated news as just one occurrence. Thus, we used only unique news for our final sample.

3. Eventually, we removed all the news on the imbalance of supply and demand before both the opening and the closing of trading time of different stock exchanges. Such news are released each trading day at the same time (usually we can see several hundreds of news coming out in a short time at the beginning and at the end of the trading sessions).

Following [8], we constructed time series of news flow intensity as follows. We divided 189-day period Δ into n non-overlapping consecutive intervals $\Delta_1, \ldots, \Delta_n$ of equal (small) longevity δ minutes, $\Delta = \Delta_1 \cup \ldots \cup \Delta_n$. We found x_t, the amount of economical and finance news reported in the world during each segment Δ_t, $t = 1, 2, \ldots, n$. The sequence $x_1, x_2, \ldots, x_n$ is the time series of news flow intensity with the δ minutes window.

Using the final data sample, we divide the whole period into non-overlapping consecutive intervals of equal length $\delta = 1$ and 5 minutes and calculated the amount of all news during each interval of time. The total amount of news released during the 189-day period is equal to

2011463. The description statistics of time series one can find in Table 1.

Table 1. Empirical properties of time series, $\delta = 1$ and $\delta =$ minutes.

δ, minutes	1	5
n	$2.7 \cdot 10^5$	$5.4 \cdot 10^4$
Mean	7.39	36.95
Minimum	0	0
Maximum	694	804
St. deviation	14.17	46.37
Median	4	24
Skewness	10.24	4.42
Kurtosis	213.47	36.83

Our time series shows that there is no clear tendency of the daily amount of news for all companies. Fig. 1 plots an example of news flow intensity with 1-min window. It is evident that time series is highly volatile and demonstrates a non-stationary behavior. Some periods have the rate of news intensity below the average (e.g. holidays, Christmas time). On the other hand, there are periods with high of news intensity, for example, in the periods of the quarterly reports and releases of the intermediate figures and earnings of companies.

4.2. Self-similarity analysis

In this subsection we present the auto-correlation and self-similarity analysis for time series of the news flow intensity using DFA and AWC.

In our empirical analysis we used the time series of the amount of news per 1 and 5 minutes as it was described in Subsection 4 4.1 from the news analytics data set containing the list of all news. Two data sets shown in Table 1 have been considered. We used DFA (of order 1) method to quantify the correlation and scaling properties of the time series.

Then, we obtained values of $F(s_i)$ for different segment lengths $s_i \in [10^1, 10^{4.5}]$ using DFA.

The problem of choosing an appropriate length for scaling range has been examined in the papers [33], [34], [35]. The impact of scaling range on the effectiveness of some detranding methods (including DFA and DMA) has been investigated in the same works. The authors find the actual scaling range for which the real value of correlation exponent γ is strictly reproduced. We used two ranges $[10^1, 10^{4.5}]$ (for 1-min data) and $[10^1, 10^4]$ (for 5-min data) in our analysis. This values belong to the scaling range recommended by these papers.

Figures 2 and 3 plot the results of applying DFA to 1-min time-series for news flow intensity.

Figures 4 and 5 present the log-log plot of dependance $F(s)$ of s for 5-min time series obtained by DFA.

It should be noted that DFA method allowed us to detect the effect of crossover described in the paper [29]. It means that the correlations of data do not hold the same scaling law for all time scales s, i.e. there is one (or maybe more) crossovers between different scaling regimes. Fig. 3 shows that for 1-min time series ($\delta = 1$) there is a difference between the values of exponent α on segment $s \in [10^1, 10^{2.5}]$ ($\alpha = 0.78$) and segment $s \in [10^{3.5}, 10^{4.5}]$ ($\alpha = 0.68$). Fig. 5 shows that for 5-min time series ($\delta = 1$) there is a difference between the values of exponent α on segment $s \in [10^1, 10^2.5]$ ($\alpha = 0.88$) and segment $s \in [10^3, 10^4]$ ($\alpha = 0.80$).

Figures 6 and 7 plot the results of applying AWC method to 1-min time-series for news flow intensity.

Figures 8 and 9 present the log-log plot of dependance $W[X](a)$ of a for 5-min time series obtained by AWC method.

The AWC method detects the crossover effect for 1-min time series ($\delta = 1$). Fig. 7 shows that there is a difference between the values of Hurst exponent H on segment $s \in [10^1, 10^{2.5}]$ ($\alpha = 0.82$) and segment $s \in [10^{3.5}, 10^{4.5}]$ ($\alpha = 0.92$). Fig. 9 shows that for 5-min time series ($\delta = 1$) the difference between the Hurst exponent H on segment $s \in [10^1, 10^2.5]$ ($H = 0.85$) and

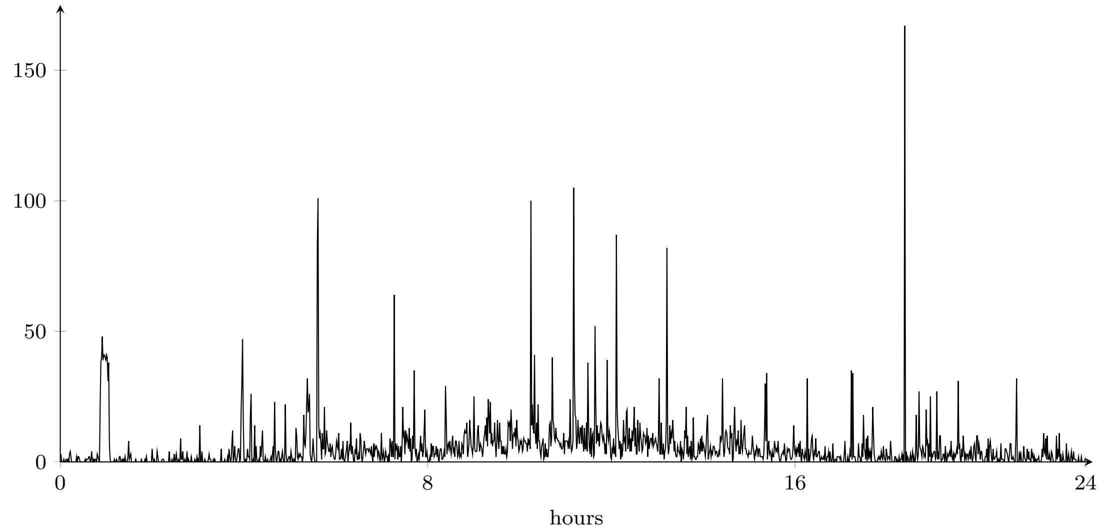

FIG. 1: Dynamics of news flow intensity (September 22, 2015).

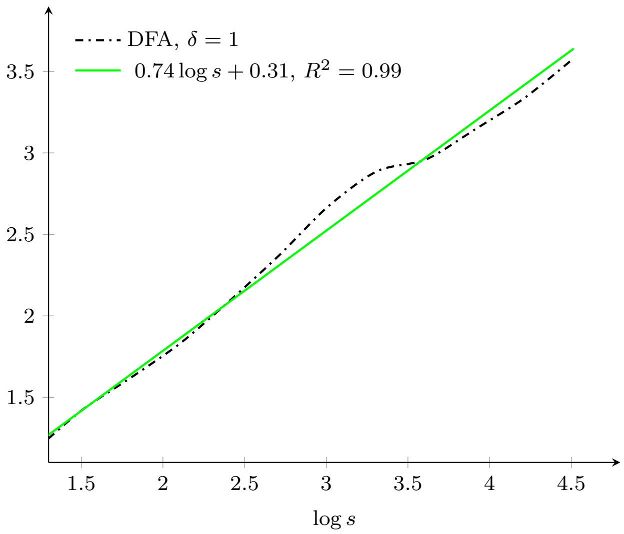

FIG. 2. $\log F(s)$ versus $\log s$ for the DFA estimation method, $\delta = 1$, with one regression. (in color)

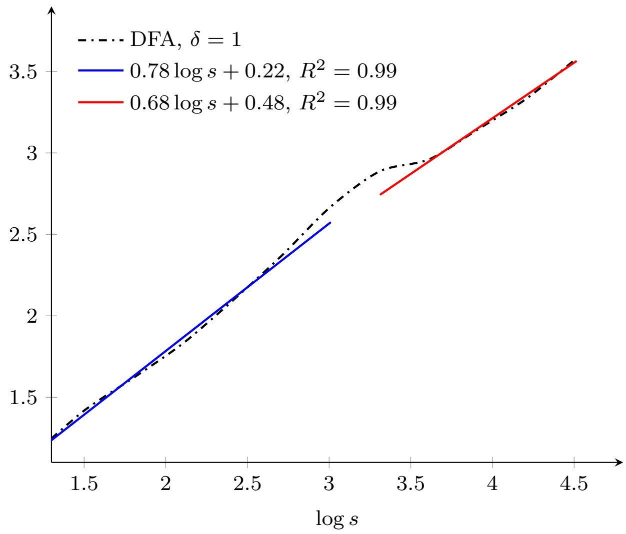

FIG. 3. $\log F(s)$ versus $\log s$ for the DFA estimation method, $\delta = 1$, with two regression. (in color)

segment $s \in [10^3, 10^4]$ ($H = 0.86$) is small.

In this paper we carry out the analysis of long-range correlations for time series of news flow intensity (with 1-min and 5-min frequencies). We estimate the Hurst exponent by two methods – DFA and AWC. Tables 2, 3 show that both methods (AWC and DFA) give almost identical estimates of the Hurst exponent on the whole scale of s both for 1 min and 5 min time series. The differences between estimates for Hurst exponent obtained by means of two different methods are less than 0.02. It should be noted that regression errors are very small and the value of R^2 is close to 1 for both methods.

We remark that both methods let us detect the so called crossover effect. Therefore, we conduct the estimation of the Hurst exponent both on small and big scales separately. It should

Table 2. Estimates of the Hurst exponent, $\delta = 1$ min

s	DFA	AWC
$s < 2^{10}$	0.783	0.819
$s > 2^{10}$	0.683	0.916
$2^3 < s < 2^{15}$	0.737	0.744

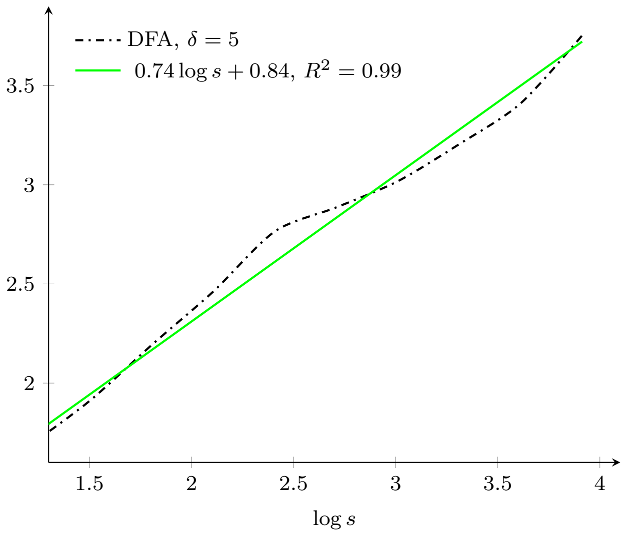

FIG. 4. $\log F(s)$ versus $\log s$ for the DFA estimation method, $\delta = 5$, with one regression.

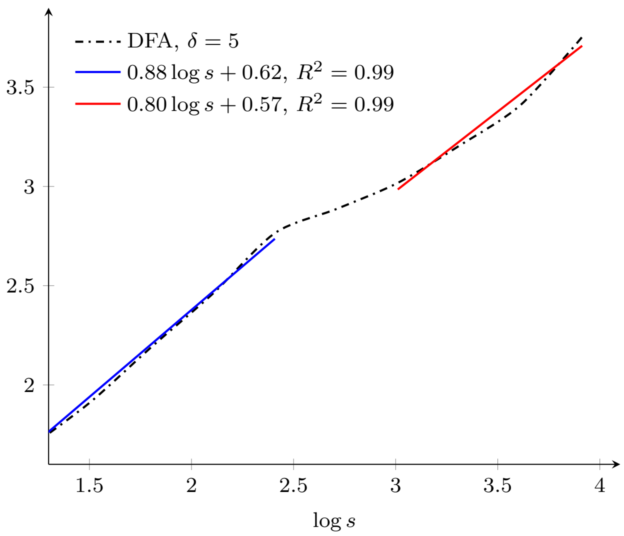

FIG. 5. $\log F(s)$ versus $\log s$ for the DFA estimation method, $\delta = 5$, with two regression.

be noted that the crossover effect matches with

Table 3. Estimates of the Hurst exponent, $\delta = 5$ min

s	DFA	AWC
$s < 2^8$	0.877	0.847
$s > 2^8$	0.801	0.86
$2^3 < s < 2^{13}$	0.738	0.718

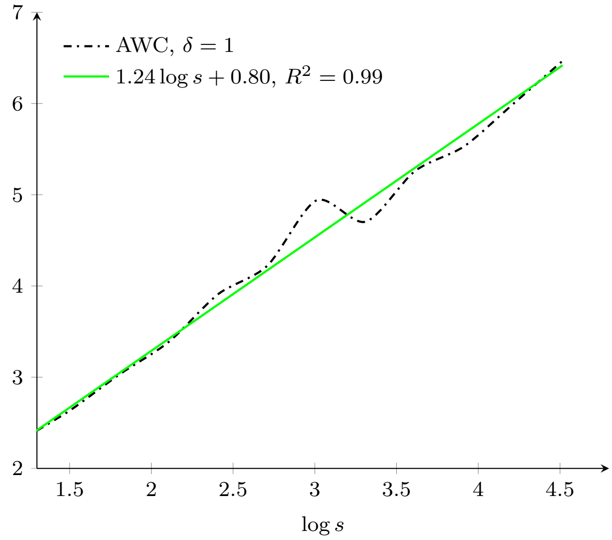

FIG. 6. $\log W[X](a)$ versus $\log a$ for AWC estimation method, $\delta = 1$, with one regression.

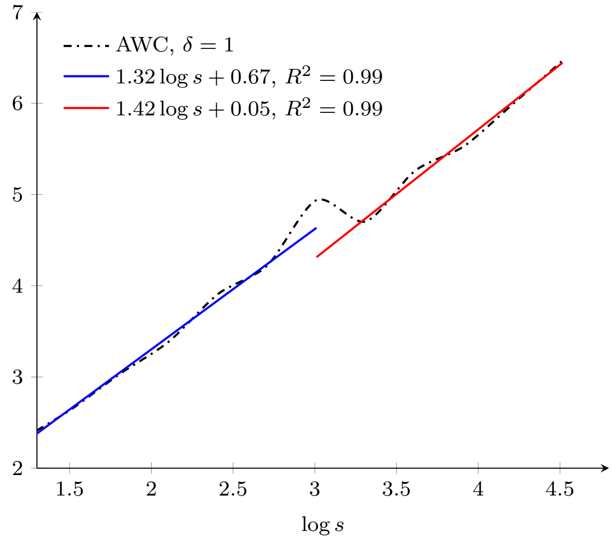

FIG. 7. $\log W[X](a)$ versus $\log a$ for AWC estimation method, $\delta = 1$, with two regression. (in color)

1 day period (or 24 hours). Estimates of the

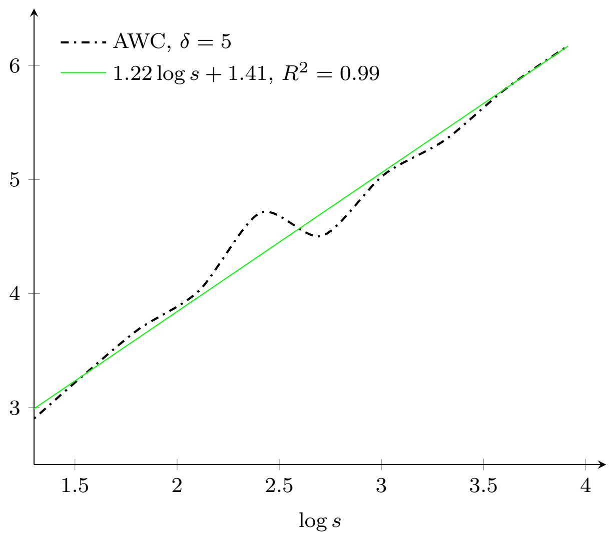

FIG. 8. $\log W[X](a)$ versus $\log a$ for AWC estimation method, $\delta = 5$, with one regression.

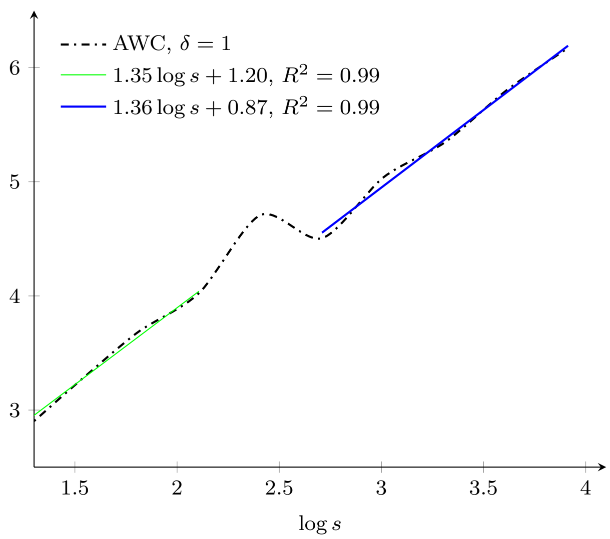

FIG. 9. $\log W[X](a)$ versus $\log a$ for AWC estimation method, $\delta = 5$, with two regression. (in color)

Hurst exponent for small scale (less than one day period) obtained by DFA and AWC are very close with differences less than 0.04 (both for 1 min and 5 min data sets). On the other hand, estimates of the Hurst exponent for bigger scales (more than one day period) are quite different (especially for 1 min data set). Estimates obtained by AWC method show that on the right hand side of crossover effect, long range correlation is becoming stronger (estimates of Hurst exponent are increasing). Contrary to that, estimates obtained by DFA method indicate a decrease in the length of the correlation. Perhaps, the DFA method gives not quite accurate estimates of the Hurst exponent while applied on large scales (to get more accurate estimates it is necessary to increase the size of the sample).

It can be concluded that both methods showed the presence of a long-range correlation in time series of news flow intensity (it follows from the fact that estimates of Hurst exponent are more than 0.5 at least on the scale $1 < \log s < 4.5$, i.e. from several minutes to dozen of days). Usage of large scales makes it possible to detect the crossover effect. Note that in this study, in addition to DFA method we also applied DFA methods of order 2 and order 3 (i.e., we detranded time series by quadratic and cubic polynomials, respectively). The results obtained by DFA methods of order 2 and order 3 were not included in the paper, since both methods have proved to be less stable compared to the DFA which uses a simple linear trend.

It should be noted that estimates of Hurst parameter in the paper [8] were obtained for the earlier period from September 1, 2010 to October 29, 2010. Due to the fact that the paper [8] showed no difference in the results for original and deseasonalized time series, in this paper we did not carry out the analysis of deseasonalized time series. Note that the empirical results of this paper fully comply with our earlier estimates of the long-range correlation parameter obtained in [8].

5. Conclusion

In this paper we examine the presence of long-range correlation of economic and financial news flow intensity using DFA and AWC. As well as the results of [8], the empirical results of this paper show that news flow time series exhibit the long-range power-law correlation. The paper shows that the behavior of long range dependence for time series of news intensity in the recent period from January 1, 2015 to September 22, 2015 did not change in comparison to the period from September 1, 2010 to October 29, 2010. Moreover, the change of the news analytics provider and the consideration of more recent data did not significantly affect the estimates of the Hurst exponent. The results show that the self-similarity property is a stable characteristic of the news information flow which serves the financial industry and stock markets.

References

[1] P. Abry, P. Flandrin, M. S. Taqqu, D. Veitch, Self-similarity and long-range dependence through the wavelet lens (2000).

[2] H. Hurst, Long-term storage capacity of reservoirs, Transactions of American Society of Civil Engineers 116 (1951) 770–799.

[3] J. Feder, Fractals, Plenum Press, New York, 1988.

[4] M. Holschneider, On the wavelet transformation of fractal objects, J. Stat. Phys. 50 (1988) 963–993.

[5] C. Peng, S. Buldyrev, S. Havlin, M. Simons, H. Stanley, A. Goldberger, Mosaic organization of DNA nucleotides, Phys. Rev. E 49 (1994) 1685–1689.

[6] E. Alessio, A. Carbone, G. Castelli, V. Frappietro, Second-order moving average and scaling of stochastic time series, Eur. Phys. J. B 27 (2002) 197–200.

[7] I. Simonsen, A. Hansen, O. M. Nes, Determination of the hurst exponent by use of wavelet transforms, Phys. Rev. E 58 (1998) 2779–2787.

[8] S. Sidorov, A. Faizliev, V. Balash, E. Korobov, Long-range correlation analysis of economic news flow intensity, Physica A: Statistical Mechanics and its Applications 444 (2016) 205 – 212.

[9] C. Peng, S. Buldyrev, A. Goldberger, S. Havlin, F. Sciortino, M. Simons, H. Stanley, Long-range correlations in nucleotide sequences, Nature 356 (1992) 168-170.

[10] S. Buldyrev, A. Goldberger, S. Havlin, R. Mantegna, M. Matsa, C. Peng, M. Simons, H. Stanley, Long-range correlation properties of coding and noncoding DNA sequences: GenBank analysis, Phys. Rev. E 51 (1995) 5084–5091.

[11] C. Peng, S. Buldyrev, A. Goldberger, R. Mantegna, M. Simons, H. Stanley, Statistical properties of DNA sequences, Physica A 221 (1995) 180–192.

[12] S. Buldyrev, N. Dokholyan, A. Goldberger, S. Havlin, C. Peng, H. Stanley, G. Viswanathan, Analysis of DNA sequences using methods of statistical physics, Physica A 249 (1998) 430–438.

[13] R.-G. Yeh, G.-Y. Chen, J.-S. Shieh, C.-D. Kuo, Parameter investigation of detrended fluctuation analysis for short-term human heart rate variability, Journal of Medical and Biological Engineering 30 (2010) 277–282.

[14] R. Acharya, C. Lim, P. Joseph, Heart rate variability analysis using correlation dimension and detrended fluctuation analysis, ITBM-RBM 23 (2002) 333-339.

[15] T. Penzel, J. W. Kantelhardt, L. Grote, J.-H. Peter, A. Bunde, Comparison of detrended fluctuation analysis and spectral analysis for heart rate variability in sleep and sleep apnea, Trans. Biomed. Eng. 50 (2003) 1143–1151.

[16] P. Castiglioni, L. Quintin, A. Civijian, G. Parati, M. D. Rienzo, Local-scale analysis of cardiovascular signals by detrended fluctuations analysis: effects of posture and exercise, Conf. Proc. IEEE Eng. Med. Biol. Soc. Ann. 29 (2007) 5035–5038.

[17] R. G. Yeh, G. Y. Chen, J. S. Shieh, C. D. Kuo, Detrended fluctuation analysis of short-term heart rate variability in late pregnant women, Auton. Neurosci-Basic Clin. 150 (2009) 122–126.

[18] K. Fukuda, L. Amaral, H. Stanley, Dynamics of temporal correlation in daily internet traffic, Global Telecommunications Conference, 2003. GLOBECOM ’03. IEEE 7 (1999) 4069 – 4073.

[19] Y. Liu, P. Cizeau, M. Meyer, C.-K. Peng, H. E. Stanley, Correlations in economic time series, Physica A: Statistical Mechanics and its Applications 245 (1997) 437 – 440.

[20] P. Cizeau, Y. Liu, M. Meyer, C.-K. Peng, H. E. Stanley, Volatility distribution in the S&P500 stock index, Physica A: Statistical Mechanics and its Applications 245 (1997) 441 – 445.
[21] M. Ausloos, N. Vandewalle, P. Boveroux, A. Minguet, K. Ivanova, Applications of statistical physics to economic and financial topics, Physica A: Statistical Mechanics and its Applications 274 (1999) 229 – 240.
[22] W. Willinger, M. S. Taqqu, V. Teverovsky, Stock market prices and long-range dependence, Finance Stochast 3 (1999) 1–139.
[23] M. Ausloos, K. Ivanova, Introducing false EUR and false EUR exchange rates, Physica A: Statistical Mechanics and its Applications 286 (2000) 353 – 366.
[24] C.-K. Peng, J. Hausdorff, S. Havlin, J. Mietus, H. Stanley, A. Goldberger, Multiple-time scales analysis of physiological time series under neural control, Physica A: Statistical Mechanics and its Applications 249 (1998) 491 – 500.
[25] J. Hausdorff, S. Mitchell, R. Firtion, C.-K. Peng, M. Cudkowicz, J. Wei, A. Goldberger, Altered fractal dynamics of gait: reduced stride-interval correlations with aging and Huntington's disease, J. Appl. Physiology 82 (1997) 262–269.
[26] R. Bryce, K. Sprague, Revisiting detrended fluctuation analysis, Scientific Reports. **2**, 315 (2012) http://dx.doi.org/10.1038/srep00315.
[27] Y.-H. Shao, G.-F. Gu, Z.-Q. Jiang, W.-X. Zhou, D. Sornette, Comparing the performance of FA, DFA and DMA using different synthetic long-range correlated time series, Scientific Reports. **2**, 835 (2012) http://dx.doi.org/10.1038/srep00835.
[28] M. S. Taqqu, V. Teverovsky, W. Willinger, Estimators for long-range dependence: An empirical study, Fractals 3 (1995) 785-798.
[29] J. W. Kantelhardt, E. Koscielny-Bunde, H. H. Rego, S. Havlin, A. Bunde, Detecting long-range correlations with detrended fluctuation analysis, Physica A: Statistical Mechanics and its Applications 295 (2001) 441 – 454.
[30] A. R. Mehrabi, H. Rassamdana, M. Sahimi, Characterization of long-range correlations in complex distributions and profiles, Phys. Rev. E 56 (1997) 712–722.
[31] I. Daubechies, Ten Lectures on Wavelets, SIAM, Philadelphia, 1992.
[32] D. B. Percival, A. T. Walden, Wavelet Methods for Time Series Analysis, Cambridge University Press, Cambridge, 2000.
[33] D. Grech, Z. Mazur, On the scaling ranges of detrended fluctuation analysis for long-term memory correlated short series of data, Physica A 392 (2013) 2384–2397.
[34] D. Grech, Z. Mazur, Scaling range of power laws that originate from fluctuation analysis, Phys. Rev. E 87 (2013) 052809.
[35] D. Grech, Z. Mazur, Impact of scaling range on the effectiveness of detrending methods, Acta Physica Polonica A 127 (3-A) (2015) 59–65.