Рассматривается задача оптимального проектирования вращающихся агрегатных станков для обработки нескольких частей.

Ключевые слова: серийная обработка; вращающиеся агрегатные станки; смешанное целочисленное программирование; генетический алгоритм; рекомбинация.

MIP-BASED HEURISTICS FOR BATCH MACHINING AT ROTARY TRANSFER MACHINES

A problem of the optimal design of a rotary transfer machine with turrets for machining multiple parts is considered. Parts are located at the loading position of rotary table in a given sequence and they are processed simultaneously on several working positions. At each working position, several machining modules can be installed to process the operations assigned to this position. They are activated sequentially or simultaneously. Constraints related to the design of machining modules, turrets, and working positions, as well as precedence constraints related to operations, are given. The problem consists in minimizing the estimated cost of the transfer machine, while reaching a given output and satisfying all the constraints. The proposed method is based on solving a sequence of subproblems generated using MIP-recombination.
Keywords: batch machining; rotary transfer machine; mixed integer programming; genetic algorithm; recombination.

INTRODUCTION

The trend in today’s market place requires flexible and adaptive manufacturing systems. A possible solution is to employ reconfigurable manufacturing systems (RMS). RMS are able to manufacture different types of products by batches without losing all other advantages of large series production systems. One of the most important problems in the design of such systems is to determine RMS configuration (i.e. numbers of machines, positions on each machine, pieces of equipment on work positions, as well as assignment of operations to pieces of equipment, sequencing of parts to be machined, etc.) and the machining modes (cutting parameters).

This paper deals with a problem of the optimal design of a rotary transfer machine with turrets for machining multiple parts. The parts are grouped in several batches which are processed sequentially. After finish of processing a current batch the rotary transfer machine is reconfigured, i.e. the fixtures of parts are changed and some spindles are mounted or dismounted if necessary. The parts of the same batch are machined simultaneously on several working positions. At each working position, several machining modules (spindle heads) can be installed to process the operations assigned to this position. They can be activated simultaneously or sequentially. Simultaneous activation is possible if machining modules are related to the different sides of the part and work in parallel. Sequential activation is realized by the use of turrets.

We consider the case when there is only one vertical turret mounted at one position or one spindle head common for all working positions. There are several horizontal spindle heads or turrets. However, there is only one horizontal spindle head or turret per position.

The considered design problem consists in the choice of orientations of parts, the partitioning of the given set of operations into positions and machining modules, and the choice of cutting modes for each spindle head and turret.

PROBLEM STATEMENT

We consider the problem of design of a rotary transfer machine with $m_0$ working positions for machining $d_0$ types of parts. The parts are grouped in $N$ batches with required output $O_v , v = 1, 2, \ldots , N$, which are processed sequentially. Parts of $v$-th batch are loaded in sequence $\pi_v = (\pi_{v1}, \pi_{v2}, \ldots, \pi_{\mu_v})$ where $\pi_{vj} \in \{0, 1, 2, \ldots, d_0\}$, $j = 1, 2, \ldots, \mu_v$, $\mu_v$ is multiple to $m_0 + 1$ and $\pi_{v0} = 0$ means that no part is loaded. Using sequences $\pi_v$ we can define in one-to-one manner function $\pi_v(i,k), i = 1, \ldots, O_v \mu_v + m_0 - 1$, of part number on the $k$-th working position after $i$ turns of the rotary table.

Let $N_d$ be the set of machining operations needed for machining of elements of the $d$-th part, $d = 1, 2, \ldots, d_0$, located on $n_d$ sides and $N^{d}_s$, $s = 1, 2, \ldots, n_d$, is a subset of operations for machining of elements of the $s$-th side of the part. The part $d$ can be located at zero position in different orientations $H(d)$ but elements of no more than one side can be machined by vertical spindle head or turret. All elements of other sides of the part have to be assigned to horizontal spindle heads or turrets. $H(d)$ can be represented by matrix of dimension $r_d \times n_d$ where $h_{rs}(d)$ is equal $j$, $j = 1, 2$, if the elements of $N^{d}_s$ can be machined by spindle head or turret of type $j$. 

1092
Let \( N = \bigcup_{d=1}^{d_0} N_d \). All operations \( p \in N \) are characterized by the following parameters: the length \( \lambda(p) \) of the working stroke for operation \( p \in N \), the range \([\gamma_1(p), \gamma_2(p)]\) of feasible values of feed rate, and the set \( H(p) \) of feasible orientations of the part (indexes \( r \in \{1, 2, \ldots, r_d\} \) of rows of matrix \( H(d) \)) for execution of operation \( p \in N_d \) by spindle head or turret of type \( j \) (vertical if \( h_r(j) = 1 \) and horizontal if \( h_r(j) = 2 \)).

Let subset \( N_k, k = 1, \ldots, m \) contain the operations from set \( N \) assigned to the \( k \)-th working position; sets \( N_{k_1} \) and \( N_{k_2} \) be the sets of operations assigned to working position \( k \) that are concerned by vertical and horizontal machining, respectively; \( b_{kj} \) be the number of machining modules (not more than \( b_{0j} \)) of type \( j \) installed at the \( k \)-th working position and respectively subsets \( N_{kj}, l = 1, \ldots, b_{kj} \) contain the operations from set \( N_{kj} \) assigned to the same machining module.

Let \( P = < P_1, \ldots, P_k, \ldots, P_m > \) is a design decision with \( P_k = (P_{1k1}, P_{2k1}, \ldots, P_{dk1}, \ldots, P_{1k1}, P_{2k1}, \ldots, P_{dk1}, \ldots, P_{2k2h1}, \ldots, P_{dk2h1}) \),
\[ P_{dk1} = (N_{dk1j}, \Gamma_{dk1j}), P_{dk1} = (N_{dk1j}) = 1, \ldots, b_{kj}) \), \( P_{dk} = (N_{dkj}) = 1, 2 \), and \( N_j = \bigcup_{d=1}^{d_0} \bigcup_{k=1}^{k_0} N_{dkj} \), \( j = 1, 2 \).

The execution time \( t^j(P_{dkj}) \) of operations from \( N_{dkj} \) with the feed per minute \( \Gamma_{dkj} \in [\max\{\gamma_1(p) \mid p \in N_{dkj}\}, \min\{\gamma_2(p) \mid p \in N_{dkj}\}] \) is equal to \( t^j(P_{dkj}) = L(N_{dkj})/\Gamma_{dkj} + \tau^a \), where \( L(N_{dkj}) = \max\{\lambda(p) \mid p \in N_{dkj}\} \), and \( \tau^a \) is an additional time for advance and disengagement of tools. We assume that if the turret of type \( j \) is installed at the \( k \)-th position then the execution time of operations from \( N_{dkj} \) is equal to \( t^j(P_{dkj}) = \tau^b_{kj} + \sum_{i=1}^{b_{kj}} t^j(P_{dkij}) \), \( j = 1, 2 \), where \( \tau^a \) is an additional time for one rotation of turret. If the spindle head is installed then \( t^j(P_{dkj}) = t^j(P_{dkij}) \), \( j = 1, 2 \). If all \( N_{dkj} \) are empty then \( t^j(P_{dkj}) = 0 \). If \( b_{kj} = 1 \) then \( t^j(P_{dkj}) = t^j(P_{dkij}) \).

The execution time \( t^j(P_{dk}) \) is defined as \( t^j(P_{dk}) = \tau^r + \max\{t^j(P_{dkij}) \mid j = 1, 2, \ldots \} \), where \( \tau^r \) is an additional time for table rotation. Then the time \( T_v(P) \) of execution of all corresponding operations after \( \mu_0 \) turns of rotary table is defined as 
\[ T_v(P) = \sum_{i=1}^{O_{\mu_0}} \max\{t^j(P_{\mu_0(i,k)}) \mid k = 1, \ldots, m_0 \} \] and the time \( T_v(P) \) for machining all the batches is equal to \( T(P) = \sum_{i=1}^{K} T_v(P) \).

We assume that the given productivity is provided, if the total time \( T(P) \) does not exceed the available time \( T_0 \).

Let \( C_1, C_2, \) and \( C_3 \) be the relative costs for one turret, one machining module of a turret, and one spindle head respectively. Since the vertical spindle head (if it presents) is common for several positions its size (and therefore the cost) depends on the number of positions to be covered. Let \( k_{min} \) and \( k_{max} \) be the minimal and the maximal position of the common vertical spindle head. Then its cost can be estimated as \( C_3 + (k_{max} - k_{min}) C_4 \) where \( C_4 \) is the relative cost for covering one additional position by vertical spindle head. If the vertical spindle turret is installed its cost can be estimated by \( C_1 + C_2 b_{h1} \). In the similar way the cost \( C(b_{k2}) \) for performing set of operations \( N_{k2} \) by associated \( b_{k2} \) machining modules can be assessed as follows:
The machine cost \( Q(P) \) is calculated as the total cost of all equipment used i.e.
\[
Q(P) = C_3 \text{sign}(|N_i|) |(1 - n_{12})| + n_{12}(C_1 + C_2 b_{k2}) + C_4 (k^{h}_{\max} - k^{h}_{\min}) + \sum_{k=1}^{m} C(b_{k2})
\]
where \( n_{12} = \sum_{k=1}^{m} \text{sign}(|N_{k12}|) \), \( \text{sign}(a) = 1 \) if \( a > 0 \), and \( \text{sign}(a) = 0 \) if \( a \leq 0 \).

The design decision \( P \) should satisfy the following constraints:
- precedence constraints which define possible sequences of operations;
- inclusion constraints which oblige to perform some pairs of operations from \( N \) at the same working position, by the same turret, by the same spindle head or even by the same spindle;
- exclusion constraints which prohibit the mutual assignment of some pairs of operations from \( N \) to the same working position, to the same turret, or to the same spindle head;
- constraints taking into account the possibility to perform operations only for certain orientations of parts;
- productivity constraints to provide the required output.

**MIP-BASED HEURISTICS**

Different types of mathematical models of the considered design problem are given in [1]. One of them is the set-theoretical model. Two other models are based on MIP formulation. The first MIP model is applied for fixed orientations of parts while the second one is capable to determine optimal orientations of parts.

There were also developed multi-start heuristics [2]. At each iteration, the heuristics creates machining modules of current position step by step. At the beginning, it builds the list \( In \) of operations, which are potentially assignable to a current machining module, taking into account precedence constraints as well as exclusion constraints on positions. The list \( In \) is modified in accordance with inclusion constraints. Then one operation or several operations with regard to inclusion constraints on machining modules and tools is chosen to be assigned to a current machining module. If it is not possible a new machining modules is created. After the assignment, the list \( In \) is modified and the assignment process is repeated. When the list \( In \) is empty or \( b_0 \) machining modules have been already created, the current position closed and productivity constraint is verified. If it is violated, the algorithm starts from the beginning (creation of the first position). The iteration is considered also unsuccessful if after creation of \( m_0 \) positions not all the operations from \( N \) are assigned.

The proposed heuristics is based on solving a sequence of MIP problems as in [3].

Let \( X_{pq} \) be decision variable which is equal to 1 if the operation \( p \) from \( N \) is assigned to the block \( q = 2(k-1)b_0 + (j-1)b_0 + l \), i.e. \( l \)-th machining module of spindle head or turret type \( j \) at the \( k \)-th position, \( B(p) \) and \( K(p) \) be sets of block indices from \( \{1, 2, \ldots, 2mb_0\} \) and of position indices from \( \{1, 2, \ldots, m_0\} \) where operation \( p \in N \) can be potentially assigned.

Based on matrices \( H(d) \), a matrix \( H \) of possible orientations of all parts can be built. Each row of \( H \) defines in one-to-one manner the orientation \( H \) and the corresponding partition of \( N \) to \( N_1 \) and \( N_2 \). For such a partition \( B(p, H) \) and \( K(p, H) \) can be calculated using algo-
rithms [4]. Then $B(p)$ and $K(p)$ are defined as $B(p) = \bigcup_{H \in \mathcal{H}} B(p,H)$ and $K(p) = \bigcup_{H \in \mathcal{H}} K(p,H)$ respectively.

Let $TR_{tot}$ be the current number of trials, $TR_{nimp}$ be the number of trials that do not improve the current solution, $N_{min}$ and $N_{max}$ be the minimal and the maximal numbers of released operations in the current candidate, $C$ be the cost of the current solution, and $C_{min}$ be the cost of the best solution. The following Algorithm is a modification of a genetic algorithm in which mutation and crossover operations are based on MIP-recombination [3].

Step 0. Generate the initial population using heuristics [2].

Step 1. Let $C_{min} = \infty$, $TR_{tot} = 0$, $TR_{nimp} = 0$.

Step 2. Choose two solutions $X'$ and $X''$ from the population.

Step 3. Let $N' = \{ p \in N | q^*(p,X') = q^*(p,X'') \}$ where $q^*(p,X) \in B(p)$ and $X_{pq}^*(p,X) = 1$.

Step 4. If $|N'| < N_{min}$ a random set is removed from $N'$. If $|N'| > N_{max}$ a random set is added to $N'$.

Step 5. Let $B(p) = \{ q^*(p,X') \}$ for $p \in N'$ and $B(p)$ be unchanged for $p \in N \setminus N'$.

Step 6. Solve the obtained MIP problem with maximal running time $T_{MIP}$.

Step 7. Compute the value $C$ of the objective function for the obtained solution.

Step 8. Replace the worst solution in population with the new solution.

Step 9. With probability $P_m$ let $N' = N \setminus N'$ and execute Steps 4–8.

Step 10. If $C_{min} > C$, then set $C_{min} = C$, $TR_{nimp} = 0$ and keep the current solution as the best, set $TR_{nimp} = TR_{nimp} + 1$ otherwise.

Step 11. Set $TR_{tot} = TR_{tot} + 1$.

Step 12. Stop if one of the following conditions holds:

- a given solution time is exceeded;
- $TR_{tot}$ is greater than the maximum number of iterations authorized;
- $TR_{nimp}$ is greater than a given value.

Go to Step 2 otherwise.

There were generated series of 100 test instances for batches of 4, 6, 8 and 10 parts. Constraints were generated using tools [5]. Experiments were carried out on ASUS notebook (1.86 Ghz, 4 Gb RAM) with academic version of CPLEX 12.2.

Table presents results for CPLEX12.2 (maximal solution time 7200 sec) with the proposed heuristics HEUR for $TR_{tot} = 200$, $TR_{nimp} = 80$, $T_{MIP} = 10$ sec, $P_m = 0$ and maximal solution time 600 sec. In this table NSOL is the number of problems with found feasible solution, NOPT is the number of problems with proven optimality, AVT is the average solution time (in sec), AVED, MIND and MAXD are average, minimal and maximal deviations (in percent’s) the found value of the objective function from the best known respectively. If no solution was found by a method, deviations were not calculated for it.

### Results for CPLEX and HEUR

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<tr>
<th>METH</th>
<th>NSOL</th>
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<th>AVT</th>
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