Анализируются поперечные плоскости изображения с компьютерной том- графией. Ребром — ограниченный контур аппроксимировался математической моделью. Задача аппроксимации решалась методом нелинейных наименьших квадратов. Были найдены оптимальные параметры модели с использованием не- линейной оптимизации. Модель полезна в регистрации изображений независимо от положения пациента на кровати и на инъекции агента радио-контраста.

Ключевые слова: компьютерная томография; метод нелинейный наимень- ших квадратов; оптимизация; моделирование.

INTRODUCTION

In medicine, many decisions on diagnostics and evaluation of effectiveness of the treatment are made by the analysis of images. Image data come from medical diagnostic techniques such as radiology, echoscopy, magnetic resonance, tomography, etc.

MODELLING THE RIBS-BOUNDED CONTOUR IN COMPUTER TOMOGRAPHY IMAGES

G. Dzemyda, M. Bilinskas

Institute of Mathematics and Informatics
Vilnius University
Vilnius, Lithuania

e-mail: gintautas.dzemyda@mii.vu.lt, mykolas.bilinskas@mii.vu.lt

The research deals with the computer tomography. The transversal plane images from computer tomography scans are analyzed. The ribs-bounded contour was approximated by some mathematical model. The problem of approximation is defined as the nonlinear least-squares one. The optimal parameters of the model were found using the nonlinear optimization. The model would be useful in registration of images independently on the patient position on the bed and on the radio-contrast agent injection.

Keywords: computer tomography; nonlinear least-squares; optimization; modelling.
Computed tomography (CT) is a technology allowing the inside of objects to be spatially viewed using computer-processed X-rays. It is very important in medical diagnostics because it shows human internal organs without cutting, e.g. brain, liver [1], prostate [2]. CT scans are 3D images, i.e. a collection of 2D images (slices), representing slices by the transversal plane. This paper deals with finding of the ribs-bounded contour. It is important for e.g. internal organ localization, because the ribs-bounded contour defines the region of location of internals in the slice. Defining the area of internal organs from the ribs-bounded contour restricts essentially the search area, where these organs are located, and may serve as the effective start for the detailed localization of particular organ. The rib anatomy is used as a reference point in CT scans analysis. In [3], an automated method is being developed in order to identify corresponding nodules in serial thoracic CT scans for interval change analysis. The problem of selection a proper function defining the ribs-bounded contour appears. In this paper, we suggest cardioid-type curve. However, it is not the only one possible. As alternative may serve snake-type curve [4]. The computing of such curve will face problems in the spine area.

The ribs-bounded contour may be important in the image registration. The image registration is a technique used to transform several images into one coordinate system. In the CT image analysis, registration would be useful e.g. in comparing slices from different CT scans of the same patient seeking to evaluate efficiency of a treatment or progress of the disease. The model of the ribs-bounded contour may be the basis of a criterion of similarity of images (slices). In this case, we will have a method of the feature-based registration. Image registration finds wide applications in medicine (eye fundus analysis [5], etc.) In the CT image analysis, an attempt to use the slice registration is done in [6]. Their comparison criteria is a vector made of white, gray and black pixels counts in a certain parts of an image. The known model of the ribs-bounded contour would make comparison of two images easier and faster.

In this paper, a mathematical model that describes the ribs-bounded contour has been developed and the problem of approximation is solved. We restrict ourselves to the slices where ribs are visible. However, this fact does not lessen the significance of our work, because many important internal organs, such as liver, heart, stomach, pancreas, lungs, are located here.

IMAGES FOR ANALYSIS

We investigate images of size $512 \times 512$, gathered by GE LightSpeed Pro 32 CT scanner. 16-bit DICOM grayscale images were obtained using it. The images were automatically linearly normalized to the interval $[0; 255]$ by window level (center) and window width HU (Hounsfield units). Fig. 1 details the possible content of particular scan slices. Depending on the slice, the heart, lungs, stomach, or liver can be seen. In both the slices of fig. 1, the internal organs are bounded by ribs. Fig. 1 was obtained after the patient was given a radiocontrast agent injection. Therefore, the heart and aorta as well as all blood vessels are bright here.

The approach below consists of two steps: extracting the bone tissue from image and approximating the ribs-bounded contour with some function. Any bone tissue extraction algorithm can be used, such as [7]. Denote $B = \{ B_i = (b_{i1}, b_{i2}) \}_{i = 1, m}$ the set of coordinates of bone pixels, where is the number of bone pixels in this slice (we skip the order number of slice in the CT scan for the simplicity).
The ribs form a shape similar to cardioid (see fig. 1):
\[
\rho(\phi) = 1 + \cos \left( \phi - \frac{\pi}{2} \right), \quad \phi \in \left[ -\frac{\pi}{2}, \frac{3\pi}{2} \right].
\] (1)

Here \( \rho \) is the radius and \( \phi \) is the polar angle. The shape of is depicted in Fig. 2 (A). It looks similar, because it features a cave which could be used to approximate ribs cave near spine. \( \frac{\pi}{2} \) is introduced in because the standard cardioid is rotated by 90° as compared with fig. 2 and the ribs-bounded contour in the images should be oriented like ribs depicted in fig. 1.

Fig. 2 indicates that a ribs-bounded contour is more condensed vertically than the standard cardioid curve. Therefore, we suggest to add optimizable parameter – power \( s \):
\[
\rho(\phi) = \left(1 + \cos \left( \phi - \frac{\pi}{2} \right) \right)^s.
\] (2)

Parameter \( s \) influences not only the vertical scale of the curve, but the form of the curve, too (see fig. 2 (B) and (C) for curves with different values of \( s \)). In the CT scan slice (fig. 1), we see a cave influenced by the breastbone. Curve is convex in this region. Therefore, we need to complement the model redefining \( \rho \) with additional member \( \rho^* \) whose form may vary depending on the cave:
\[
\rho(\phi) = \left(1 + \cos \left( \phi - \frac{\pi}{2} \right) \right)^s - c\rho^*(\phi).
\] (3)

This member realizes the cave by subtraction of some value from the right side of starting from \( \phi = \frac{\pi}{2} - \beta \) till \( \phi = \frac{\pi}{2} + \beta \). \( \beta \) is an angle, defining the region of subtraction. The non-negative multiplier \( c \) defines the scale of subtraction, if \( c = 0 \), we get the case with no subtraction.

The member \( \rho^*(\phi) \) depends on \( \phi \) with domain \( \left[ -\frac{\pi}{2}, \frac{3\pi}{2} \right] \) and has special properties. It must (a) be unimodal non-negative function on \( \phi \) and achieve the maximal value as
\( \varphi = \frac{\pi}{2} \), (b) be symmetrical function in respect of \( \varphi = \frac{\pi}{2} \), (c) be equal to 0 when \( \varphi = \frac{\pi}{2} - \beta \) and \( \varphi = \frac{\pi}{2} + \beta \), (d) have zero first and second derivatives on \( \varphi \) when \( \varphi = \frac{\pi}{2} - \beta \) and \( \varphi = \frac{\pi}{2} + \beta \). Function \( \rho^+ \) may be as follows:

\[
\rho^+(\varphi) = \begin{cases} 
\sin \left( \frac{\pi(\varphi - \frac{\pi}{2} + \beta)}{2\beta} \right), & \text{if } \beta \geq \left| \varphi - \frac{\pi}{2} \right| \\
0, & \text{otherwise}
\end{cases}
\]  

In and, we have three control parameters characterizing the breastbone cave for which the optimal values need to be found: \( \beta \) is an angle, defining the region of subtraction, \( l \) defines the steepness of curve describing the cave, \( c \) is the scale of subtraction. Moreover, we need some additional parameters \( a \) and \( b \) that define the horizontal and vertical scales of the curve that approximates the rib-bounded contour, respectively. The curve should be fitted among ribs in the picture of bone tissue. For this reason, we need the optimal place of the point of corresponding to \( \varphi = 0 \) in the picture; denote coordinates of this point by \( (x_0, y_0) \).

As we see in fig. 1, the rib-bounded contour has some rotation in respect to the bed. Therefore, we should introduce the angle \( \theta \) of such rotation.

If the values of \( s, \theta, a, b, x_0, y_0, \beta, c, l \) are fixed, we can draw some parametric curve \((x, y) = (x(\varphi), y(\varphi))\) approximating the rib-bounded contour:

\[
x(\varphi) = x_0 + a \varphi \cos \varphi \cos \theta - b \varphi \sin \varphi \sin \theta, \\
y(\varphi) = y_0 + a \varphi \cos \varphi \sin \theta - b \varphi \sin \varphi \cos \theta.
\]  

While the ribs are symmetrical, their cross-section may appear asymmetrical due to the position of the patient on the bed. Moreover, we observe spinous process in the spine bone tissue (see fig. 1). This spinous process influences the shape of the model as well. The notices above lead to the addition of a line segment parallel to sagittal axis in the model. This line segment must include at least all the spine in the slice: from \((x_0; y_0)\) till the bottom of the unrotated model (case \( \theta = 0 \)). Formally, the line segment is a part of a line that is bounded by two distinct end points: \((x_0; y_0)\) and \((x_0 + (y_0 - \min_y \sin \theta); y_0 - (y_0 - \min_y \cos \theta)\) where \( \min_y \) is the minimal value of \( y \) defined by in case of \( \theta = 0 \): \( \min_y = \min_{\varphi} (y_0 + b \varphi \sin \varphi) \).

**DETAILS ON THE REALIZATION AND CONSTRAINTS**

If \( \varphi \) runs through the interval \([-\frac{\pi}{2}; \frac{3\pi}{2}]\) with a step \( \frac{1}{n} \), we get sequence \( C' = \left\{(x_i, y_i) : i = 0, n-1\right\}\) of the curve points, where \( x_i = x \left( \frac{2\pi i}{n} \right) \) and \( y_i = y \left( \frac{2\pi i}{n} \right) \), \( x_i \) and \( y_i \) are defined by. We use \( n = 180 \). The line segment above is sampled by sequence of \( n_i \) points \( C'' = \left\{ C''_i = \left( x_0 + \frac{(y_0 - \min_y \sin \theta) \sin \varphi}{n_i} i, y_0 - \frac{(y_0 - \min_y \cos \theta) \sin \varphi}{n_i} i \right), i = 1, n_i \right\} \). Both the sequences together form the sequence \( C = (C', C'') \) of the length \( n + n_i \). \( \min_y \) is approximated...
by \( \min_{i=0,n} \left( y_0 + b \left( \frac{2 \pi}{n} i \right) \sin \left( \frac{2 \pi}{n} i \right) \right) \) to avoid the analytical solving of equality to zero the derivative of \( y(\varphi) \) on \( \varphi \).

Optimizing unconstrained model could result in strange shapes like fig. 3, a and 3, b, this occurs when \( c \geq 2^\varphi \). In fact \( c \) should not come near \( 2^\varphi \), so we restrict it to be less than \( 1 \). Approximation of the ribs-bounded contour is put on the image. Experiments have showed that optimization algorithm may catch strange local minima (fig. 3, c), constraint \( \max_y \left( 2 a \varphi(\varphi) \cos \varphi \right) > \max_y \left( b \varphi(\varphi) \sin \varphi \right) - \min_y \left( b \varphi(\varphi) \sin \varphi \right) \) (it means width > height) can prevent it. The model may be flipped over, if a constraint \( \theta \) is unconstrained (see fig. 3, d), so we restrict \( -\pi/6 < \theta < \pi/6 \).

![Fig. 3. A shape of the model when \( c > 2^\varphi \) (a), \( c = 2^\varphi \) (b), height >> width (c) and \( \theta \approx \pi \) (d)](image)

**OPTIMIZATION PROBLEM**

The model of ribs-bounded contour of particular slice has nine parameters whose values can be varied seeking to find the best approximation of the contour: \( s, \theta, a, b, x_0, y_0, \beta, c, l \). The optimal values of parameters must be defined by the set \( B = \{ B_i = (b_{i1}, b_{i2}), i = 1, m \} \) of coordinates of bone pixels obtained during the analysis of CT image slices. The optimization problem to find optimal \( s, \theta, a, b, x_0, y_0, \beta, c \) and \( l \) is formulated as a least square one:

\[
\min_{s, \theta, a, b, x_0, y_0, \beta, c, l} f(s, \theta, a, b, x_0, y_0, \beta, c, l),
\]

\[
f() = \sum_{i=1}^{m} \| B_i - C_k \|^2, \quad k_i = \arg \min_{j=0,n} \| B_i - C_j \|.
\] (6)

Local optimization method should be used to solve. We use the Matlab realization of the quasi-Newton method.

The experiments have shown that the optimization is more reliable when the problem is divided into two stages, firstly the model is fitted without breastbone cave: \( \min_{s, \theta, a, b, x_0, y_0, \pi, 0, 2} f(s, \theta, a, b, x_0, y_0, 0, 2) \) (\( \beta, c, l \) values are fixed), then solving problem from such starting point, where \( s, \theta, a, b, x_0, y_0 \) are gathered from the first suproblem and \( \beta = \pi, c = 0, l = 2 \).

Also the experiments have shown that optimal values of parameter \( \beta \) are close to \( \pi \). Therefore, \( \beta \) can be fixed at \( \pi \), i.e. it may be set as non-optimizable parameter. This means that the subtrahend for breastbone is subtracted from \( \rho(\varphi) \) for all \( \varphi \in \left[ -\pi/2, 3\pi/2 \right] \), and the
shape of the breastbone cave is controlled by scale $c$ and power $l$ only. The model in polar coordinates becomes $\rho(\varphi) = (1 + \cos(\varphi - \pi/2))^l - c \sin\left(\frac{\varphi + \pi/2}{2}\right)$.

Examples of slices with the approximating curve is shown in fig. 4.

![Fig. 4. Examples of fitting](image)

CONCLUSIONS

In this paper, a method for the analysis of transversal plane images of computer tomography scans is presented. This method allows us not only to find the approximation of the ribs-bounded contour, but also to evaluate the patient rotation around the vertical axis during scans.

The proposed approximation defines the rib-bounded contour exactly. The average distance between the model the bone-tissue pixels is about 4.53 mm. The model can be applied to any 2D slice, where the ribs are visible. Further research can be directed at the CT scan image registration. When evaluating the effectiveness of the treatment, pre- and post-treatment CT scans must be made (for the same patient) and compared by aligning (registering) these two (or more) scans [6]. The model of the ribs-bounded contour can serve as the basis of a criterion of similarity of images (slices).

LITERATURE