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Quantum Mechanical Scalar Particle with Intrinsic Structure in External Magnetic and Electric Fields: Influence of Geometrical Background

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Relativistic theory of the Cox's scalar not point-like particle is developed in the presence of electromagnetic and gravitational fields. This theory is specified in simple geometrical backgrounds: Euclid's, Lobachevsky's, and Riemann's. Wave equations for the Cox's particle, relativistic and non-relativistic, are solved exactly in the presence of external uniform magnetic and electric fields in the case of Minkowski space. Non-trivial additional structure of the particle modifies the frequency of a quantum oscillator arising effectively in the presence of an external magnetic field. Extension of these problems to the case of hyperbolic Lobachevsky space is examined. In the presence of a magnetic field, the quantum problem in radial variable has been solved exactly; quantum motion in z-direction is described by 1-dimensional Schrödinger-like equation in an effective potential which turns out to be too difficult for analytical treatment. In the presence of an electric field, the situation is similar. The same analysis has been performed for spherical Riemann space model. General conclusion can be done: the role of large scale structure of the Universe depends greatly on the form of basic equations for a particle, any modification of them lead to new physical phenomena due to non-Euclidean geometry background.

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We start with general covariant Proca-like system for the Cox's scalar particle [1] with additional intrinsic structure

$$K_{\rho}{}^{\alpha}\left(i\partial_{\alpha}-\frac{e}{c\hbar}A_{\alpha}\right)\Phi=\frac{mc}{\hbar}\Phi_{\rho},$$
$$\left(\frac{i}{\sqrt{-g}}\frac{\partial}{\partial x^{\alpha}}\sqrt{-g}-\frac{e}{c\hbar}A_{\alpha}\right)g^{\alpha\beta}\Phi_{\beta}=\frac{mc}{\hbar}\Phi \quad (1)$$

where $K_{\rho}{}^{\alpha}$ is a tensor inverse to

$$\Lambda_{\sigma}{}^{\alpha}=mc\delta_{\sigma}{}^{\alpha}+\lambda F_{\sigma}{}^{\alpha}\,;$$

λ is the Cox's parameter related with non-trivial structure of the particle;

$$K_{\rho}{}^{\alpha}=\lambda_1\,\delta_{\alpha}{}^{\beta}+\lambda_2\,F_{\alpha}{}^{\beta}$$
$$+\lambda_3\,F_{\alpha}{}^{\rho}F_{\rho}{}^{\beta}+\lambda_4\,F_{\alpha}{}^{\rho}F_{\rho}{}^{\sigma}F_{\sigma}{}^{\beta}\,, \quad (2)$$

four coefficients λ_i are expressed through invariants of an external electromagnetic field.

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In geometrical models of the following type $dS^2 = c^2dt^2 + g_{kl}(x)\ dx^k dx^l$, one can perform non-relativistic approximation [2] and derive extended Schrödinger-like equation. In particular, for external magnetic and electric fields this generalized wave equation is of the form shown below.

The Scrödinger equation for the Cox's particle in a magnetic field has the form

$$D_t\ \Psi = -\frac{1}{2m}\ \overset{\circ}{D}_k\ g^{kj}(x)\ \overset{*}{D}_j\ \Psi \tag{3}$$

where one uses the following notations:

$$\frac{i\hbar}{\sqrt{-g}}\frac{\partial}{\partial x^k}\sqrt{-g} - eA_k = c\ \overset{\circ}{D}_k\ ,$$

$$\overset{*}{D}_1 = K_1{}^l D_l = \frac{1}{1+\Gamma^2 B_i B^i}$$

$$\times\left[D_1 + \Gamma(B_2D^3 - B_3D^2) + \Gamma^2 B^1(B_iD_i)\right],$$

$$\overset{*}{D}_2 = K_2{}^l D_l = \frac{1}{1+\Gamma^2 B_i B^i}$$

$$\times\left[D_2 + \Gamma(B_3D^1 - B_1D^3) + \Gamma^2 B^2(B_iD_i)\right],$$

$$\overset{*}{D}_3 = K_3{}^l D_l = \frac{1}{1+\Gamma^2 B_i B^i}$$

$$\times\left[D_3 + \Gamma(B_1D^2 - B_2D^1) + \Gamma^2 B^3(B_iD_i)\right]$$

$$g^{22}g^{33}B_1 = B^1\ ,\ g^{33}g^{11}B_2 = B^2\ ,$$

$$g^{11}g^{22}B_3 = B^3\ ,\ F_{ij} = \epsilon_{ijk}B_k\ ;$$

$\Gamma = \lambda/mc)$

The Schrödinger equation for the Cox's particle in an electric field is

$$\left(D_t - c\frac{\Gamma^2 E_iE^imc + \Gamma E^jD_j}{2(1+\Gamma^2E_iE^i)}\right)\Psi$$

$$= -\frac{1}{2m}\ \overset{\circ}{D}_k\ g^{kj}\left[D_j + \frac{\Gamma^2E_j(E^iD_i) + mc\Gamma E_j}{1+\Gamma^2E_iE^i}\right]\Psi \tag{4}$$

where

$$g^{11}E_1 = E^1,\ g^{22}E_2 = E^2,\ g^{33}E_3 = E^3,$$
$$E_i = F_{0i},\ A_\alpha = (A_0, 0, 0, 0).$$

The Schrödinger equation for such a particle in a uniform magnetic field for Minkowski space leads to the following radial equation

$$\left[\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{(m-br^2)^2}{r^2} + \Lambda\right]R(r) = 0 \tag{5}$$

where the notation is used

$$\Lambda = \epsilon - (1-\eta^2)k^2 + 2\eta b,$$
$$\epsilon = \frac{2mE}{\hbar^2}(1+\gamma^2)\ . \tag{6}$$

The formula for the energy levels due to the non-trivial structure of a particle is

$$E = \frac{p^2}{2M} + \frac{\Omega\hbar}{1-(\Gamma B)^2}\left(n + \frac{m+\mid m\mid+1}{2}\right) - \frac{\Omega\hbar}{1-(\Gamma B)^2}\frac{\Gamma B}{2}, \tag{7}$$

so the intrinsic structure of the Cox's particle modifies the frequency of the quantum oscillator

$$\Omega = \frac{eB}{Mc} \quad\Longrightarrow\quad \tilde{\Omega} = \frac{\Omega}{1-(\Gamma B)^2}\ . \tag{8}$$

Now, we consider the Cox's particle in a magnetic field for the Lobachevsky space geometry. In cylindrical coordinates in the Lobachevsky space, analogue of a uniform magnetic field is determined by the relations (in dimensionless coordinates)

$$dS^2 = c^2dt^2 - \mathrm{ch}^2 z(dr^2 + \mathrm{sh}^2 r d\phi^2) + dz^2,$$
$$A_\phi = -B\rho^2(\mathrm{ch}\ r - 1),$$
$$B_3 = -B\rho\ \mathrm{sh}\ r, \qquad B^3 = -\frac{B}{\rho\ \mathrm{sh}\ r\ \mathrm{ch}^4 z},$$
$$B_iB^i = B^2\mathrm{ch}^{-4}z. \tag{9}$$

After separation of the variables we obtain the radial equation for $R(r)$:

$$\left(\frac{d^2}{dr^2} + \frac{\mathrm{ch}r}{\mathrm{sh}r}\frac{d}{dr} - \frac{[m - b(\mathrm{ch}\ r - 1)]^2}{\mathrm{sh}^2 r} + \Lambda\right)R = 0, \tag{10}$$

and the equation for $Z(z)$:

$$\left(\frac{d^2}{dz^2} + 2\frac{\text{sh } z}{\text{ch } z}\frac{d}{dz} + \epsilon + \frac{b\gamma - \Lambda \text{ ch}^2 z}{\text{ch}^4 z - \gamma^2}\right) Z = 0. \tag{11}$$

The radial problem leads to the finite series of bound states

$$\rho^2\Lambda_0 - \frac{1}{4} = 2\frac{eB}{\hbar c}\rho^2(\frac{m+ \mid m \mid}{2} + n + 1/2)$$
$$-(\frac{m+ \mid m \mid}{2} + n + 1/2)^2, n = 0, 1, \dots, N_B. \tag{12}$$

In equation for $Z(z)$, let us eliminate the first derivative term:

$$Z = \frac{1}{\text{ch } z}f(z), \quad U(z) = -\frac{b\gamma - \Lambda \text{ ch}^2 z}{\text{ch}^4 z - \gamma^2},$$
$$\left(\frac{d^2}{dz^2} + \epsilon - 1 - U(z)\right) f(z) = 0. \tag{13}$$

Eq. (13) can be viewed as the Schrödinger equation with rather complicated effective potential field $U(z)$ (a bell-shaped barrier with one point of local extremum at z=0) In the variables $\cosh^2 z = y$, the differential equation $\tilde{(}13)$ reads

$$\left[\frac{d^2}{dy^2} + \left(\frac{3}{2}\frac{1}{y} + \frac{1}{2}\frac{1}{y-1}\right)\frac{d}{dy} + \frac{\epsilon}{4y(y-1)} + \frac{b\gamma - \Lambda\, y}{(y-\gamma)(y+\gamma)4y(y-1)}\right] Z(y) = 0\ . \tag{14}$$

Note that the three singular points $y = 0, \pm\gamma$ ($|\gamma| << 1$) are located outside the physical range of the variable. The equation in z variable leads us to a problem of reflecting the particle from the bell-shaped barrier created effectively by the geometry of Lobachevsky space and the intrinsic structure of the particle.

Two problems detailed above are quite typical to characterize the influence of an intrinsic structure of a Cox's particle of its behavior in external electromagnetic and gravitational fields.

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