

SOME REPRESENTATIONS OF FINITE GROUPS

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We consider the arithmetic background of integral representations of finite groups over the maximal orders of local and algebraic number fields.

Some infinite series of integral pairwise inequivalent absolutely irreducible representations of finite p -groups with the extra congruence conditions are constructed. Certain problems concerning integral two-dimensional representations over number rings are discussed.

In his recent publication [9] J.-P. Serre emphasized remarkable connections between integral irreducible representations of the group of quaternions and genus theory of Gauss and Hilbert, and the theory of Hilbert's symbol. This was also considered in our recent paper [7] as an application to the description of globally irreducible representations over arithmetic rings which was earlier introduced by F. Van Oystaeyen and A. E. Zalesskii, see [8]. This is also motivated by the following question considered by J.-P. Serre, W. Feit and other mathematicians (see also [1, 2, 6, 9]):

Let $\rho : G \rightarrow GL_n(K)$ be a linear representation of a finite group G over a number field K . Is it possible to realize ρ over O_K , the ring of integers of K , i. e. is ρ conjugate to a homomorphism of G into $GL_n(O_K)$?

Another approach to generalization of integral representations of finite groups was proposed by D. K. Faddeev in [3] (see also [4] and [5]) where a generalization of the theory of Steinitz and Chevalley has been suggested.

References

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