Thermal Convection in a
Ferromagnetic Fluid in
an Inhomogeneous
Magnetic Field

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Dimensionless ratios describing heat transfer in a ferromagnetic fluid are introduced. Conditions under which magnetic-field perturbations can be neglected are derived. It is shown on the basis of numerical and experimental results that ferromagnetic fluids can be used for controlled cooling of current-carrying conductors.

\[ \frac{\partial \theta}{\partial t} + (\vec{v} \cdot \nabla) \theta = -\frac{1}{\rho} \nabla \cdot (\vec{v} \Theta) - \nabla \cdot \left[ \mu_\theta \nabla \Theta \right], \quad \nabla \cdot \vec{v} = 0, \quad \nabla \cdot \vec{B} = 0, \quad \frac{\partial \Theta}{\partial x} = 0, \quad \frac{\partial \vec{v}}{\partial t} = 0, \quad \nabla \cdot \vec{B} = 0, \quad \vec{B} = \nabla \times \vec{A}, \quad \vec{A} = \nabla \phi - \frac{\mu_0 M_0}{\mu_0 M_0 + \mu_r M_r} \vec{H}, \]

where

\[ \phi = \frac{\mu_0 M_0}{\mu_0 M_0 + \mu_r M_r} \left( \frac{\mu_r M_r}{\mu_0 M_0 + \mu_r M_r} \right) \frac{\mu_0 M_0}{\mu_0 M_0 + \mu_r M_r} \left( \vec{H} \cdot \vec{H} \right) \]

\[ \Theta = \frac{T - T_c}{T_c - T_c} \]

\[ \mu_0 M_0 = \mu_0 M_0 + \mu_r M_r \]

The physical quantities are the same as in [2]. The scale factors used are: for the coordinate, the characteristic dimension of the cavity \( \ell \); for the time, \( \ell^2 / \nu \); for velocity, \( \ell / \nu \); for the pressure, \( \rho \ell^2 / \nu \); for the
temperature, \( \theta \); and for the magnetic field \( \mathbf{G} \). The equation of state for magnetization was taken in the form

\[
M = M^0 - X(\tau - \tau^0) + X(\mu - \mu^0).
\]

The dimensionless equations (1)-(3) describe convective heat transfer in an incompressible ferromagnetic fluid. Quantities \( Gr, \, Pr, \, Gr_m, \, A_1 \), \( A_2 \) and \( A_3 \) describe convective heat transfer in ferromagnetic fluids and have the following meaning. \( Gr \) and \( Pr \) are the standard Grashof and Prandtl numbers; \( Gr_m \) is the magnetic Grashof number, defining the onset of convective instability brought about by a purely magnetic mechanism. A comparison of \( Gr \) and \( Gr_m \) makes it possible to estimate the contributions of the gravitational and the magnetic mechanisms to convection. The dimensionless quantity \( A_3 \) is the ratio of the inhomogeneity of the magnetic field, produced by nonisothermicity of the ferromagnetic fluid, to the magnitude of the characteristic magnetic field gradient. Ratio \( A_2 \) expresses the degree of inhomogeneity of the magnetic field, while ratio \( A_1 \) describes the deviation of the magnetization equation of state from linearity.

It is seen by analyzing Eqs. (1)-(3) that the temperature can be neglected in the Maxwell equation if

\[
A_3 \ll 1 \quad (4)
\]

or

\[
A_3 \ll A_1, \, A_2 \quad \text{and} \quad A_3 \ll A_1 \quad (5)
\]

When conditions (4) and (5) are satisfied, the hydrodynamic equations can be solved independently of the Maxwell equations. This approximation is termed inductionless in magnetohydrodynamics.

Let us consider, in this approximation, convective motion in a ferromagnetic fluid in a finite vertical layer of height \( h \) and width \( \ell \), the sides of which are maintained at constant but different temperatures, while a linear temperature profile is specified on its horizontal faces. It is assumed that the heated vertical boundary of the layer can be treated as a thin, infinitely long conducting slab, which conducts a current \( I \), uniformly distributed over the width \( h \).

The above boundary layer problem was solved numerically by a monotonic second-order conservative difference scheme [3]. The components of the magnetic field of the actively conducting plate were calculated from familiar formulas [4], with \( \ell \) as the characteristic dimension and the value of the magnetic-field gradient on the central axis of the plate as the characteristic gradient.

Figure 1 depicts the effect of the magnetic field of the actively conducting plate on convective heat transfer in a ferromagnetic fluid in a cavity with square cross section, i.e., with dimensions \( h/\ell \gg 1 \).

The effect of the magnetic field is insignificant at \( Ra_m, \ Gr_m \ll 0.2 \, Ra, \)

and convective heat transfer is governed by gravity forces. When \( Ra \) and
\( \text{Ra}_m \) are in the above ratio, heat transfer through the layer is quite satisfactorily described by the dimensionless equation

\[
N_u = 0.126 \left( \frac{v}{\ell} \right)^{0.25} \text{Ra}_m^{0.28},
\]

which holds in the ranges \( \text{Ra} > 5 \times 10^9 \), \( \text{Ra}_m < 0.2 \text{Ra} \) 1 \( \leq \text{h/\ell} \leq 10 \).

In the range \( 0.2 \text{Ra} < \text{Ra}_m < 10 \text{Ra} \), the effect of magnetic forces is greater. But, whereas at \( \text{Ra} < 10^4 \) the Nusselt number increases rapidly with \( \text{Ra}_m \), in this range, at large \( \text{Ra} \) (i.e., \( \text{Ra} = 10^5 \)) the thermal convection is slightly less intensive, because of a specific interaction of the gravitational and magnetic mechanisms of induction of convective motion in the transitional region.

A further increase in the current in the plate at \( \text{Ra}_m > 10^2 \text{Ra} \) produces a merger of all heat transfer curves, as magnetic forces become controlling. As the field becomes more inhomogeneous, the contribution of gravity-induced convection to convective heat transfer becomes smaller and, tending to zero at the limit. At \( \text{Ra}_m > 5 \times 10^3 \), \( 1 \leq \text{h/\ell} \leq 10 \), the limiting curve may be approximated by the dimensionless equation

\[
N_u = 0.224 \left( \frac{v}{\ell} \right)^{0.25} \text{Ra}_m^{0.28},
\]

which is represented by the dashed curve in Fig. 1.

The thermal convection of ferromagnetic fluids was investigated in an annular layer, formed by an external cylinder held at constant temperature and an inner cylinder through which direct current (0 to 500 A)
was used. We used a ferromagnetic fluid with the following principal properties: $Q' = 1234 \text{ kg/m}^3$, volumetric concentration of magnetic material 8.3\%, saturation magnetization $M_s = 2.1 \cdot 10^4 \text{ A/m}$, $\zeta = 8 \cdot 10^{-3}$ N·sec/m², $\lambda = 0.18 \text{ W/m·K}$, $Pr = 97$. Under these conditions Eqs. (4) and (5) are satisfied. Hence the contribution of the magnetic convection mechanism to heat transfer was determined in terms of the magnetic Rayleigh number, in which the characteristic quantities were $G = \frac{2}{\delta(\delta c)}$, $\ell = \Delta c$, $\beta = \frac{\Delta T}{\Delta T}$.

Figure 2 presents the heat transfer data as a plot of $Nu = Nu(Ra)$ at various currents; for comparison, this graph also presents the experimental results of other workers. The data are correlated in the form of $Nu = Nu(Ra_m)$ for several constant values of $Ra$ (Fig. 3). It can be seen that when the current attains values corresponding to $Ra_m = Ra$, the heat transfer rate increases significantly. Starting with the inflection point the heat transfer curves can be described by the empirical expression

$$Nu = L \left( \frac{Ra}{\ell} \right)^{-0.1} (Ra_m)^{0.63}_m,$$

where $L$ is a function of $Ra$. The curve of $L = L(Ra)$ is plotted in Fig. 4.
It was shown as a result of numerical and experimental studies that automatic heat transfer control becomes possible when active conductors are cooled by ferromagnetic liquids, since the removal rate of the released heat increases with the current through the conductor.

NOMENCLATURE

$\Delta r = r_2 - r_1$ - difference in the radii of cylinders forming the annular layer;

$\Delta T$ - temperature difference between cylinders.

REFERENCES


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