# The Modelling of the Displacements of the Fang in the Bone Stock Under the Concentrated Force 

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#### Abstract

This paper is devoted to the modelling of the displacements of tooth root in bone stock. These displacements appear under stress creating by the orthodontic appliance. The shape of tooth root is defined by the equations of elliptical hyperboloids. The calculations and visualization of displacements field are realized for two clinical cases of point load action on single-rooted premolar and canine tooth. Analytical transforms, calculations, imaging of the displacements of fang are performed with Mathematica.


## 1 Introduction

Research of biomechanical influence of orthodontic appliances on teeth roots is one of the factors of successful treatment of dentofacial anomaly. Review of some results obtained in this direction using finite element method is presented in monograph [1]. In particular, the finite element modeling of orthodontic teeth movement under first and second order arc bending are presented here using two- and three dimensional models. Review of other literature sources shows that for calculation of teeth roots displacements appears under point load and moment action on a tooth produced by orthodontic appliances the finite element method is also generally used. Papers [2, 3] demonstrated the finite element calculation of four incisors movement under moment loading of lateral incisor. Computation of initial tooth movements and angles of rotation of canine tooth was also done. Analysis of force acting on teeth and provide given displacements is carried out in mono-graph [4] based on solution of equations system of statics and kinematics. In this monography angles and forces of activation, coordinates of connecting points between arc and wireframe of orthodontic appliances and also forces of biomechanical reactions are obtained. A lot of literatures about finding optimal value of load for necessary teeth movement are presented in work [5]. The author shows until now there are no concrete recommendations about it because of difficulties in representation of three-dimensional tooth movement. One of the ways of three-dimensional teeth roots displacement representation with shape of elliptical hyperboloid is presented in the work [6].

Particularly here teeth roots movement in periodont based on expressions describing infinitely small displacements of rigid bodies in elastic medium. Present study develop this actual direction and devoted to analysis of tooth root movement in bone stock appears under point load action on a tooth. Displacements calculation
carried out taking into account atrophy of bone stock. In this case the atrophy is a vertical resorption of alveole bone stock [1]. Taking into account the atrophy is necessary because tolerance of tooth after horizontal and vertical resorption essentially decreases and the tooth movement increase under action of load produced by orthodontic appliances.

## 2 Bone Stock Stiffness and Centers of Strength

Let us assume that tooth root is a rigid body which shape is defined by expression of complex elliptical hyperboloid in the following way:

$$
\begin{equation*}
F\left(x_{1}, x_{2}, x_{3}\right)=x_{3}-\frac{H}{\sqrt{1+p^{2}}-p}\left(\sqrt{\frac{x_{1}^{2}}{a_{k}^{2}}+\frac{x_{2}^{2}}{b^{2}}+p^{2}}-p\right)=0, \tag{1}
\end{equation*}
$$

where $H$ is height of the fang; $p$ is parameter which define the rounding of crest; $a_{k}=a_{1}$ when $x \geq 0$ and $a_{k}=a_{2}$ when $x<0, a_{1}, a_{2}, b$ are half-axles of ellipse of root ; $p$ is the thickness of periodont; surface $F_{0}\left(x_{1}, x_{2}, x_{3}\right)$ bounds the surface of periodont from the bone stock, surface $F\left(x_{1}, x_{2}, x_{3}\right)$ bounds the surface of periodont from the fang.

Represent elastic displacements $u_{1}, u_{2}$ and $u_{3}$ in the region of bone stock adjoining to tooth root in the way allows taking into account unlimited decreasing of displacements away from tooth root:

$$
\begin{array}{r}
u_{1}=\frac{H\left(u_{1}^{(0)}+\varphi_{2}\left(x_{3}-x_{3}^{(A)}\right)-\varphi_{3} x_{2}\right)}{H-F\left(x_{1}, x_{2}, x_{3}\right)}, \\
u_{2}=\frac{H\left(u_{2}^{(0)}+\varphi_{3}\left(x_{1}-x_{1}^{(B)}\right)-\varphi_{1}\left(x_{3}-x_{3}^{(B)}\right)\right)}{H-F\left(x_{1}, x_{2}, x_{3}\right)},  \tag{2}\\
u_{3}=\frac{H\left(u_{3}^{(0)}+\varphi_{1} x_{2}-\varphi_{2}\left(x_{1}-x_{1}^{(C)}\right)\right)}{H-F\left(x_{1}, x_{2}, x_{3}\right)},
\end{array}
$$

Here $u_{k}^{(0)}$ are translational displacements of root along coordinate axis; $\varphi_{k}$ are angles of rotation of tooth root along coordinate axis, $k=\overline{1,3} ; x_{1}^{(B)}, x_{1}^{(C)}, x_{3}^{(A)}$ and $x_{3}^{(B)}$ are strength centers coordinates of tooth root. On the tooth root surface $F\left(x_{1}, x_{2}, x_{3}\right)=0$ displacements $u_{1}, u_{2}$ and $u_{3}$ coincides with rigid body displacements. It is significant, the resistance centers are points $A\left(0,0, x_{3}^{(A)}\right), B\left(x_{1}^{(B)}, 0, x_{3}^{(B)}\right)$ and $C\left(x_{1}^{(C)}, 0,0\right)$. Through these points pass lines of action of two horizontal and one vertical forces. Under action of these forces tooth root is obtained only translational displacements. The line of force action passing through point $A$ is parallel to coordinate axis $x_{1}$, passing through point $B$ is parallel to coordinate axis $x_{2}$ and passing through point $C$ is parallel to coordinate axis $x_{3}$.

Translational displacements and angles of rotation of tooth root can be finding from equilibrium conditions (principal vector and principal moment of all forces are equal to zero and all stresses on the surface of tooth root are also equal to zero) [7]:

$$
\begin{equation*}
\int_{F}(\vec{n} \cdot \sigma) d F-\vec{P}=0, \int_{F} \vec{r} \times(\vec{n} \cdot \sigma) d F-\vec{m}=0 \tag{3}
\end{equation*}
$$

where $\vec{m}=\left(m_{1}, m_{2}, m_{3}\right)$ is the principal moment of the external forces; $\vec{P}=$ $\left(P_{1}, P_{2}, P_{3}\right)$ is the principal vector of the external forces; $\vec{r}$ is the radius-vector drawn from the corresponding resistance center; $\vec{n}=\left(n_{1}, n_{2}, n_{3}\right)$ is the unit normal to surface $F\left(x_{1}, x_{2}, x_{3}\right)=0, n_{k}=\frac{1}{\Delta} \frac{\partial F}{\partial x_{k}}, \Delta=\sqrt{\sum_{k=1}^{3}\left(\frac{\partial F}{\partial x_{k}}\right)^{2}} ; \sigma$ is the stress tensor, $\sigma_{i j}=G\left(\frac{\partial_{i} u_{j}}{\partial_{j} u_{i}}+\frac{\nu \delta_{i j}}{1-\nu} \sum_{k=1}^{3} \frac{\partial u_{k}}{\partial x_{k}}\right), G=\frac{E}{2(1+\nu)}, G$ is the module of shear, $E$ is the module of elasticity, $\nu$ is the Poisson coefficient, $\delta_{i j}=1$, if $i=j, \delta_{i j}=0$, if $i \neq j$.

After putting expressions (2) into expressions (3) and some simplifications the following formulas are obtained:

$$
\begin{array}{r}
u_{k}^{(0)}=\frac{P_{k}}{c_{k}} \\
\varphi_{1}=\frac{\left(P_{2} x_{1}^{(f)}-P_{1} x_{2}^{(f)}\right) \mu_{13}+\left(P_{2} x_{3}^{(f)}-P_{3} x_{2}^{(f)}\right) \mu_{3}}{\mu_{13}^{2}-\mu_{1} \mu_{3}}, \\
\varphi_{2}=\frac{P_{1} x_{3}^{(f)}-P_{3} x_{1}^{(f)}}{\mu_{2}},  \tag{4}\\
\varphi_{3}=\frac{\left(P_{2} x_{1}^{(f)}-P_{1} x_{2}^{(f)}\right) \mu_{1}+\left(P_{2} x_{3}^{(f)}-P_{3} x_{2}^{(f)}\right) \mu_{13}}{\mu_{1} \mu_{3}-\mu_{13}^{2}},
\end{array}
$$

Here $x_{k}^{(f)}$ is coordinates of point load action. Also the following notations in the expressions (4) are used: $c_{k}$ is the stiffness of bone stock under translational displacements of tooth root along $0 x_{k}$ coordinate axis (numerically equal to force causing the tooth root movement equals to 1 mm along $0 x_{k}$ axis), $\mu_{k}$ is the stiffness of bone stock under rotation of tooth root relatively $0 x_{k}$ coordinate axis (numerically equal to moment of forces which is necessary apply to tooth to rotate it relatively $0 x_{k}$ axis through an angle $\varphi_{k}=1$ ), $\mu_{13}$ is the stiffness of bone stock under rotation of tooth root relatively $0 x_{3}$ coordinate axis under action of moment relatively $0 x_{1}$ axis (numerically equal to moment of forces which is necessary apply to tooth root relatively $0 x_{1}$ axis to rotate it relatively $0 x_{3}$ axis through an angle $\varphi_{3}=1$ ). Expressions for stiffness $c_{k}, \mu_{k}$ and $\mu_{13}$ can be finding after integration over the surface $F\left(x_{1}, x_{2}, x_{3}\right)=0$ the following equations:

$$
\begin{gather*}
c_{k}=G H \int_{F} g_{i} \frac{d F}{\Delta}, g_{i}=\frac{1}{m} \sum_{k=1}^{3}\left(\left(\gamma \delta_{i k}+1\right)\left(\partial_{k} F\right)^{2}-\left(\partial_{i} F\right)^{2}\right),  \tag{5}\\
m=\left(H-F\left(x_{1}, x_{2}, x_{3}\right)\right)^{2}, \gamma=\frac{2(1-\nu)}{1-2 \nu}, i \neq j \neq k=\overline{1,3}, \\
\mu_{1}=G H \int_{F}\left(g_{2}\left(x_{3}-x_{3}^{(B)}\right)^{2}+g_{3} x_{2}^{2}+2(1-\gamma) x_{2}\left(x_{3}-x_{3}^{(B)}\right) \partial_{2} F \partial_{3} F\right) \frac{d F}{\Delta},  \tag{6}\\
\mu_{2}=G H \int_{F}\left(g_{1}\left(x_{3}-x_{3}^{(A)}\right)^{2}+g_{3}\left(x_{1}-x_{1}^{(C)}\right)^{2}+\right.  \tag{7}\\
\left.+2(1-\gamma) \partial_{1} F \partial_{3} F\left(x_{1}-x_{1}^{(C)}\right)\left(x_{3}-x_{3}^{(A)}\right)\right) \frac{d F}{\Delta},
\end{gather*}
$$

$$
\begin{array}{r}
\mu_{3}=G H \int_{F}\left(g_{2}\left(x_{1}-x_{1}^{(B)}\right)^{2}+g_{1} x_{2}^{2}+2(1-\gamma) x_{2}\left(x_{1}-x_{1}^{(B)}\right) \partial_{1} F \partial_{2} F\right) \frac{d F}{\Delta}, \\
\mu_{13}=\mu_{31}=-G H \int_{F}\left(\left(x_{1}-x_{1}^{(B)}\right)\left(x_{3}-x_{3}^{(B)}\right)\left(\partial_{3} F\right)^{2}+\right. \\
+(1-\gamma) x_{2} \partial_{3} F\left(\left(x_{1}-x_{1}^{(B)}\right) \partial_{2} F-x_{2} \partial_{1} F\right)+  \tag{9}\\
\left.+\left(x_{3}-x_{3}^{(B)}\right)\left(\left(x_{1}-x_{1}^{(B)}\right)\left(\left(\partial_{1} F\right)^{2}+\gamma\left(\partial_{2} F\right)^{2}\right)+(1-\gamma) x_{2} \partial_{1} \partial_{2}\right)\right) \frac{d F}{\Delta}
\end{array}
$$

where $r_{1}=\left(\sqrt{1+p^{2}}-p\right)^{2}, r_{2}=\ln \left(\frac{1}{p}+1\right), \partial_{k} F=\frac{\partial F}{\partial x_{k}}$. For an atrophy consideration during integration of expressions (5)-(9) let us assume the equations for representation of ellipse axles $a_{k}$ and $b$ on tooth root surface and also the height of root which is located in the bone stock in the following way:

$$
\begin{array}{r}
a_{k}=a_{0 k} \sqrt{s\left(s+2 p(1-s)\left(\sqrt{1-p^{2}}-p\right)\right)}, \\
b=b_{0} \sqrt{s\left(s+2 p(1-s)\left(\sqrt{1-p^{2}}-p\right)\right)},  \tag{10}\\
H=H_{0} s,
\end{array}
$$

where $a_{0 k}, b_{0}$ are ellipse axles limiting tooth root section profile without atrophy, $H_{0}$ is height of tooth root without atrophy, $s$ is parameter characterized the height of bone stock which connect to periodont $(0 \leq s \leq 1)$.

Expressions for strength centers coordinates determination can be finding after equating to zero coefficients before angles of rotation in the first group of equilibrium equations. After some modification we will get:

$$
\begin{array}{r}
x_{3}^{(A)}=\frac{c_{13}\left(s_{133}+s_{31}\right)-c_{3}\left(s_{311}+s_{13}\right)}{c_{13}^{2}-c_{1} c_{3}}, \\
x_{1}^{(B)}=\frac{s_{122}+s_{21}}{c_{2}}, x_{3}^{(B)}=\frac{s_{322}+s_{23}}{c_{2}},  \tag{11}\\
x_{1}^{(C)}=\frac{c_{13}\left(s_{311}+s_{13}\right)-c_{1}\left(s_{133}+s_{31}\right)}{c_{13}^{2}-c_{1} c_{3}},
\end{array}
$$

where $c_{i j}=c_{j i}=\frac{G(1-\gamma)}{h_{0}} \int_{F} \partial_{i} F \partial_{j} F \frac{d F}{\Delta}, s_{i j}=\frac{G}{h_{0}} \int_{F} x_{j} g_{i} \frac{d F}{\Delta}, s_{i j k}=\frac{G(1-\gamma)}{h_{0}} \int_{F} x_{k} \partial_{i} F \partial_{j} F \frac{d F}{\Delta}$, $s_{j i i}=\frac{G(1-\gamma)}{h_{0}} \int_{F} x_{i} \partial_{j} F \partial_{i} F \frac{d F}{\Delta}, i \neq j \neq k=\overline{1,3}$.

By pitting equations (4)-(11) into formulas (2) expressions for displacements vector components which depends from geometrical parameters of tooth root, value, direction and coordinates of point load action and also from elastic properties of bone stock are obtained. Using these expressions displacements $u_{k}\left(x_{1}, x_{2}, x_{3}\right), k=\overline{1,3}$ of tooth root under action of point load and moment can be finding.

## 3 Displacements Calculation

Let's do calculation of tooth root displacements for different clinical cases. One of them is the case of tooth movement under horizontal load action which is necessary
for the following prosthesis. Figure 1 shows the result of visualization of tooth root location before and after movement. Geometrical dimensions of tooth root (single-rooted premolar) describes by the following parameters $a_{1}=a_{2}=5, b=3.5$, $H=14.3 \mathrm{~mm}, p=0.4$ [8]. The Poisson's ratio for bone stock $\nu=0.25$, modulus of elasticity $E=16.1 \mathrm{GPa}$ [9]. The value of load $P=1 \mathrm{kN}$ [11], positive direction of load describes by the directional cosines $\cos \left(\alpha_{1}\right)=\cos \left(\alpha_{3}\right)=0, \cos \left(\alpha_{2}\right)=-1\left(\alpha_{k}\right.$ is angle between force vector and axis $\left.x_{k}, k=\overline{1,3}\right)$. The load point coordinates is $(0,3,14.3) \mathrm{mm}$.

The other widespread clinical case connects to the elimination of aesthetic defect which is the result of maxilla canine movement. At the same time it is recommend to patient to eliminate one of the proximate teeth and canine movement oral and vertically down. Let's take the following dimensions for canine root: $a_{1}=2, a_{2}=5$, $b=4, H=15.7 \mathrm{~mm}, p=0.5$ [8]. The value of load $P=1 \mathrm{kN}$ [11], positive direction of load describes by the directional cosines $\cos \left(\alpha_{1}\right)=\frac{1}{2}, \cos \left(\alpha_{2}\right)=0, \cos \left(\alpha_{3}\right)=\frac{\sqrt{3}}{2}$. The load point coordinates is $(-5,0,15.7) \mathrm{mm}$.

Results of maximum teeth roots displacements calculation are as follows $u_{\max }=$ $\sqrt{\sum_{i=1}^{3} u_{\text {imax }}^{2}}$ and for canine are $u_{\max }=5.76 \mathrm{mkm}$, where $\mathrm{s}=1 \quad(\mathrm{H}$-Bone $=100)$; $u_{\max }=10.55 \mathrm{mkm}$, where $\mathrm{s}=3 / 4(\mathrm{H}$-Bone $=75) ; u_{\max }=24.27 \mathrm{mkm}$, where $\mathrm{s}=1 / 2$ ( H -Bone $=50$ ); and for premolar are $u_{\max }=7.76 \mathrm{mkm}$, where $\mathrm{s}=1$ (H-Bone $=100$ ); $u_{\max }=14.45 \mathrm{mkm}$, where $\mathrm{s}=3 / 4(\mathrm{H}$-Bone $=75) ; u_{\max }=33.62 \mathrm{mkm}$. where $\mathrm{s}=1 / 2$ (H-Bone=50). These results are obtained with Mathematica.

In figures 1 and 2 visualization of tooth root location before and after load action for premolar and canine are presented. The visualization is constructed with Mathematica. For these two cases the symbol of point load action on the tooth root is showed in figures. Scaling of movement is realized because of small displacements in comparison with dimensions of teeth roots.

It is important for illustration of premolar root location after load action displacements are increased in 50 times, for illustration of canine root location displacements are increased in 100 times. Big displacements of premolar root in comparison with root of canine under action of the same load can be explained by the less stiffness of bone stock under translational displacements and rotation of premolar root [10].

## 4 Conclusions

Obtained expressions can be used in practice by orthodontist for realization of calculation experiment for finding the value and direction of displacements in the case of action on tooth root of point load and moment, particularly from pin of orthodontic appliance. This evaluation allows to measure out load and define it direction correctly taking into account the bone stock atrophy. It is significant that obtained explicit formulas for displacements can be used for development of computer program which allows to render of teeth roots displacement before and after movement.


Figure 1: Position of premolar root before (1) and after (2) displacement under parameter atrophy $s=\frac{1}{2}$


Figure 2: Position of canine root before (1) and after (2) displacement under parameter atrophy $s=\frac{1}{2}$

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